

# Quantum Graphology

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## **Abstract.**

We review quantum chaos on graphs. We construct a unitary operator which represents the quantum evolution on the graph and study its spectral and wavefunction statistics. This operator is the analogue of the classical evolution operator on the graph. It allow us to establish a connection between the corresponding periodic orbits and the statistical properties of eigenvalues and eigenfunctions. Specifically, for the energy-averaged spectral form factor we derived an exact combinatorial expression which illustrate the role of correlations between families of isometric orbits. We also show that enhanced wave function localization due to the presence of short unstable periodic orbits and strong scarring can rely on completely different mechanisms.

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## **1. Introduction**

Quantum graphs have recently attracted a lot of interest [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. A special volume containing a number of contributions can be found in [18]. The attention is due to the fact that quantum graphs can be viewed as typical and simple examples for the large class of systems in which classically chaotic dynamics implies universal spectral correlations in the semiclassical limit [20, 21]. Up to now we have only a very limited understanding of the reasons for this universality. In a semiclassical approach to this problem the main stumbling block is the intricate interference between the contributions of (exponentially many) periodic orbits [22, 23]. Using quantum graphs as model systems it is possible to pinpoint and isolate this central

problem. In graphs, an exact trace formula exists which is based on the periodic orbits of a mixing classical dynamical system [1, 24]. Moreover the orbits can be specified by a finite symbolic code with Markovian grammar. Based on these simplifications it is possible to rewrite the spectral form factor or any other two-point correlation functions in terms of a combinatorial problem [1, 2, 6, 9]. This combinatorial problem on graphs has been solved with promising results: It was shown that the form factor, ensemble averaged over graphs with a single non-trivial vertex and two attached bonds (2-Hydra) coincides exactly with the random-matrix result for the  $2 \times 2$  CUE [2]. A simple algorithm which can evaluate the resulting combinatorial sum for any graph was presented in [9]. In [3] the short-time expansion of the form factor for  $v$ -Hydra graphs (i. e. one central node with  $v$  bonds attached) was computed in the limit  $N \rightarrow \infty$ . In [6] a periodic-orbit sum was used to prove Anderson localization in an infinite chain graph with randomized bond lengths. In [8] the form factor of binary graphs was shown to approach the random-matrix prediction when the number of vertices increases. In [11] the second order contribution  $-2\tau^2$  to the form factor, was derived and was shown to be related to correlations within pairs of orbits differing in the orientation of one of the two loops resulting from a self-intersection of the orbit. Finally, in [16] a field theoretical method was used to evaluate exactly the form factor of large graphs. Very recently, the spectral properties of quantum graphs were studied experimentally by the Warsaw group [17] who constructed a microwave graph network.

The transport properties of open quantum graphs were also investigated quite thoroughly. In [7] compact graphs were connected with leads to infinity and was shown that they display all the features which characterize quantum chaotic scattering. In [14] the open quantum graphs were used to calculate shot-noise corrections while in [19] the same system was employed in order to understand current relaxation phenomena from open chaotic systems.

Quite recently the interest on quantum graphs moved towards understanding statistical properties of wavefunctions. In [15] the statistics of the nodal points was analyzed, while in [10, 13] quantum graphs were used in order to understand scarring of quantum eigenstates. A scar is a quantum eigenfunction with excess density near an unstable classical periodic orbit (PO). Such states are not expected within Random-Matrix Theory (RMT), which predicts that wavefunctions must be evenly distributed over phase space, up to quantum fluctuations [25]. Experimental evidence and applications of scars come from systems as diverse as microwave resonators [26], quantum wells in a magnetic field [27], Faraday waves in confined geometries [28], open quantum dots [29] and semiconductor diode lasers [30].

This contribution, is structured in the following way. In the following Section 2, the main definitions and properties of quantum graphs are given. We concentrate on the unitary bond-scattering matrix  $U$  which can be interpreted as a quantum evolution operator on the graph. Section 3 deals with the corresponding classical dynamical system. In Section 4, the statistical properties of the eigenphase spectrum of the bond-scattering matrix  $U$  are analyzed and related to the periodic orbits of the classical

dynamics. Scarring phenomenon is discussed and analyzed in Section 5. Finally, our conclusions and outlook are summarized in the last Section 6.

## 2. Quantum Graphs: Basic Facts

We start with a presentation and discussion of the Schrödinger operator for graphs. Graphs consist of  $V$  vertices connected by  $B$  bonds. The *valency*  $v_i$  of a vertex  $i$  is the number of bonds meeting at that vertex. The graph is called *v-regular* if all the vertices have the same valency  $v$ . When the vertices  $i$  and  $j$  are connected, we denote the connecting bond by  $b = (i, j)$ . The same bond can also be referred to as  $\vec{b} \equiv (Min(i, j), Max(i, j))$  or  $\overleftarrow{b} \equiv (Max(i, j), Min(i, j))$  whenever we need to assign a direction to the bond. A bond with coinciding endpoints is called a *loop*. Finally, a graph is called *bipartite* if the vertices can be divided into two disjoint groups such that any vertices belonging to the same group are not connected.

Associated to every graph is its *connectivity (adjacency) matrix*  $C_{i,j}$ . It is a square matrix of size  $V$  whose matrix elements  $C_{i,j}$  are given in the following way

$$C_{i,j} = C_{j,i} = \begin{cases} 1 & \text{if } i, j \text{ are connected} \\ 0 & \text{otherwise} \end{cases}.$$

For graphs without loops the diagonal elements of  $C$  are zero. The connectivity matrix of connected graphs cannot be written as a block diagonal matrix. The valency of a vertex is given in terms of the connectivity matrix, by  $v_i = \sum_{j=1}^V C_{i,j}$  and the total number of undirected bonds is  $B = \frac{1}{2} \sum_{i,j=1}^V C_{i,j}$ .

For the quantum description we assign to each bond  $b = (i, j)$  a coordinate  $x_{i,j}$  which indicates the position along the bond.  $x_{i,j}$  takes the value 0 at the vertex  $i$  and the value  $L_{i,j} \equiv L_{j,i}$  at the vertex  $j$  while  $x_{j,i}$  is zero at  $j$  and  $L_{i,j}$  at  $i$ . We have thus defined the *length matrix*  $L_{i,j}$  with matrix elements different from zero, whenever  $C_{i,j} \neq 0$  and  $L_{i,j} = L_{j,i}$  for  $b = 1, \dots, B$ . The wave function  $\Psi$  contains  $B$  components  $\Psi_{b_1}(x_{b_1}), \Psi_{b_2}(x_{b_2}), \dots, \Psi_{b_B}(x_{b_B})$  where the set  $\{b_i\}_{i=1}^B$  consists of  $B$  different undirected bonds.

The Schrödinger operator (with  $\hbar = 2m = 1$ ) is defined on a graph in the following way: On each bond  $b$ , the component  $\Psi_b$  of the total wave function  $\Psi$  is a solution of the one-dimensional equation

$$\left(-i\frac{d}{dx} - A_b\right)^2 \Psi_b(x) = k^2 \Psi_b(x). \quad (1)$$

We included a “magnetic vector potential”  $A_b$  (with  $\Re e(A_b) \neq 0$  and  $A_{\vec{b}} = -A_{\overleftarrow{b}}$ ) which breaks the time reversal symmetry. In most applications we shall assume that all the  $A_b$ ’s are equal and the bond index will be dropped. On each of the bonds, the general solution of (1) is a superposition of two counter propagating waves

$$\Psi_{b=(i,j)} = a_{i,j} e^{i(k+A_{i,j})x_{i,j}} + a_{j,i} e^{i(k+A_{j,i})x_{j,i}} \quad (2)$$

The coefficients  $a_{i,j}$  form a vector  $\mathbf{a} \equiv (a_{b_1}, \dots, a_{b_B}, a_{b_1}^*, \dots, a_{b_B}^*)^T$  of complex numbers which uniquely determines an element in a  $2B$ -dimensional Hilbert space. This space corresponds to "free wave" solutions since we did not yet impose any conditions which the solutions of (1) have to satisfy at the vertices.

The most general boundary conditions at the vertices are given in terms of unitary  $v_j \times v_j$  vertex-scattering matrices  $\sigma_{l,m}^{(j)}(k)$ , where  $l$  and  $m$  go over all the vertices which are connected to  $j$ . At each vertex  $j$ , incoming and outgoing components of the wave function are related by

$$a_{j,l} = \sum_{m=1}^{v_j} \sigma_{l,m}^{(j)}(k) e^{ikL_{jm}} a_{m,j}, \quad (3)$$

which implies current conservation. The particular form

$$\sigma_{l,m}^{(j)} = \frac{2}{v_j} - \delta_{l,m} \quad (4)$$

for the vertex-scattering matrices was shown in [1] to be compatible with continuity of the wave function and current conservation at the vertices. (4) is referred to as *Neumann* boundary conditions. Bellow, we will concentrate on this type of graphs. Moreover we will always assume fully connected graphs i.e. the valency is  $v_j = v = V - 1$ ,  $\forall j = 1, \dots, V$ .

Stationary states of the graph satisfy (3) at each vertex. These conditions can be combined into

$$\mathbf{a} = U(k) \mathbf{a}, \quad (5)$$

such that the secular equation determining the eigenenergies and the corresponding eigenfunctions of the graph is of the form [1]

$$\det [I - U(k, A)] = 0. \quad (6)$$

Here, the unitary *bond-scattering matrix*

$$U(k, A) = D(k; A) T \quad (7)$$

acting in the  $2B$ -dimensional space of directed bonds has been introduced. The matrices  $D$  and  $T$  are given by

$$\begin{aligned} D_{ij,i'j'}(k, A) &= \delta_{i,i'} \delta_{j,j'} e^{ikL_{ij} + iA_{i,j}L_{ij}}; \\ T_{ji,nm} &= \delta_{n,i} C_{j,i} C_{i,m} \sigma_{j,m}^{(i)}. \end{aligned} \quad (8)$$

$T$  contains the topology of the graph and is equivalent to the complete set of vertex-scattering matrices, while  $D$  contains the metric information about the bonds. Hereafter, the bond lengths  $L_m$  ( $m = 1, \dots, B$ ) will be chosen to be incommensurate in order to avoid non-generic degeneracies.

It is instructive to interpret the action of  $U$  on an arbitrary graph state as its time evolution over an interval corresponding to the mean bond length of the graph such that

$$\mathbf{a}(t) = U^t \mathbf{a}(0), \quad t = 0, 1, 2, \dots \quad (9)$$

Clearly the solutions of (5) are stationary with respect to this time evolution.  $n$  in (9) represents a discrete (topological) time counting the collisions of the particle with vertices of the graph. In this "picture" the diagonal matrix  $D_{mn}(k) = \delta_{mn} e^{ikl_m}$  describes the free propagation along the bonds of the network while  $T$  assigns a scattering amplitude for transitions between connected directed bonds. As we will see in the next section it specifies a Markovian random walk on the graph which is the classical analogue of Eq. (9).

### 3. Periodic orbits and classical dynamics on graphs

In this section we discuss the classical dynamics corresponding to the quantum evolution (9) implied by  $U$ . To introduce this dynamics we employ a Liouvillian approach, where a classical evolution operator assigns transition probabilities in a phase space of  $2B$  directed bonds [1]. If  $\rho_b(t)$  denotes the probability to occupy the (directed) bond  $b$  at the (discrete) topological time  $t$ , we can write down a Markovian Master equation of the form

$$\rho_b(t+1) = \sum_{b'} M_{b,b'} \rho_{b'}(t). \quad (10)$$

The classical (Frobenius-Perron) evolution operator  $M$  has matrix elements

$$M_{ij,nm} = \delta_{j,n} P_{i \rightarrow m}^{(j)} \quad (11)$$

with  $P_{ji \rightarrow ij'}^{(i)}$  denoting the transition probability between the directed bonds  $b = (j, i)$  and  $b' = (i, j')$ . To make the connection with the quantum description, we adopt the quantum transition probabilities, expressed as the absolute squares of matrix elements of  $M$

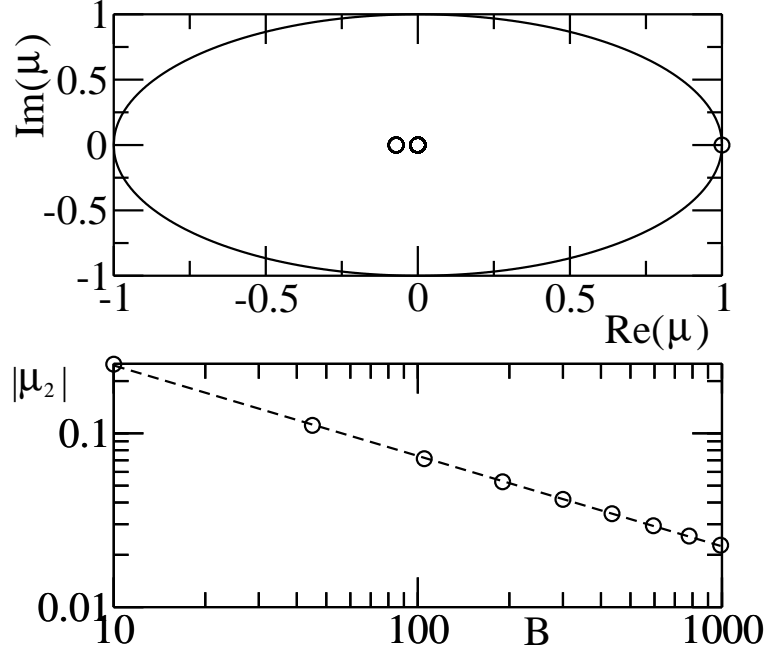
$$P_{j \rightarrow j'}^{(i)} = \left| \sigma_{jj'}^{(i)}(k) \right|^2. \quad (12)$$

Note that  $P_{j \rightarrow j'}^{(i)}$  and  $M$  do not involve any metric information on the graph.

The unitarity of the bond-scattering matrix  $U$  guarantees  $\sum_{b=1}^{2B} M_{b,b'} = 1$  and  $0 \leq M_{b,b'} \leq 1$ , so that the total probability that the particle is on any bond remains conserved during the evolution. The spectrum of  $M$ , denoted as  $\{\mu_b\}$  with  $b = 1, \dots, 2B$ , is restricted to the interior of the unit circle and  $\mu_1 = 1$  is always an eigenvalue with the corresponding eigenvector  $|1\rangle = \frac{1}{2B} (1, 1, \dots, 1)^T$ . In most cases, the eigenvalue 1 is the only eigenvalue on the unit circle. Then, the evolution is ergodic since any initial density will evolve to the eigenvector  $|1\rangle$  which corresponds to a uniform distribution (equilibrium). The rate at which equilibrium is approached is determined by the gap to the next largest eigenvalue. If this gap exists, the dynamics is also mixing.

It was shown recently [16] that mixing dynamics alone does not suffice to guarantee universality of the spectral statistics of quantum graphs ‡. An additional condition proven recently by Gutzmann and Altland [16] states that in the limit of  $B \rightarrow \infty$ , the

‡ For an example of a mixing graph with non-universal spectral statistics, see [3]







**Figure 1.** Upper panel: The spectrum of the classical evolution operator  $M$  for the case of  $V = 15$  fully connected graph. Lower panel: The scaling of the second maximum eigenvalue  $|\mu_2|$  with respect to  $B$ . Solid line is the best linear fitting indicating that  $|\mu_2| \propto B^{-0.5}$ .

spectral gap has to be constant or at least vanish slowly enough as  $\Delta_g \equiv (1 - |\mu_2|) \propto B^{-\alpha}$  with  $0 \leq \alpha < 0.5$  and  $\mu_2$  being the second maximum eigenvalue of  $M$ . In Fig. 1 we report our numerical results for Neumann fully connected graphs. We see that this type of graph satisfies the condition requested by [16].

Graphs are one dimensional and the motion on the bonds is simple and stable. Ergodic (mixing) dynamics is generated because at each vertex a (Markovian) choice of one out of  $v$  directions is made. Thus, chaos on graphs originates from the multiple connectivity of the (otherwise linear) system [1].

Despite the probabilistic nature of the classical dynamics, the concept of a classical orbit can be introduced. A classical orbit on a graph is an itinerary of successively connected directed bonds  $(i_1, i_2), (i_2, i_3), \dots$ . An orbit is *periodic* with period  $t_p$  if for all  $k$ ,  $(i_{t_p+k}, i_{t_p+k+1}) = (i_k, i_{k+1})$ . For graphs without loops or multiple bonds, the sequence of vertices  $i_1, i_2, \dots$  with  $i_m \in [1, V]$  and  $C_{i_m, i_{m+1}} = 1$  for all  $m$  represents a unique code for the orbit. This is a finite coding which is governed by a Markovian grammar provided by the connectivity matrix. In this sense, the symbolic dynamics on the graph is Bernoulli. This analogy is strengthened by further evidence: The number of  $t_p$ -PO's on the graph is  $\frac{1}{t_p} \text{tr} C^{t_p}$ , where  $C$  is the connectivity matrix. Since its largest eigenvalue  $\Gamma_c$  is bounded between the minimum and the maximum valency i.e.  $\min v_i \leq \Gamma_c \leq \max v_i$ , periodic orbits proliferate exponentially with topological entropy  $\approx \log \Gamma_c$ .

$t_p$	$p$	$M_p$	$\Lambda_p (v \rightarrow \infty)$
2		$(2/v - 1)^4$	$4/v$
3		$(2/v)^6$	$2 \ln(v)$
4		$(2/v)^4 (2/v - 1)^4$	$\ln(v)$
		$(2/v)^8$	$2 \ln(v)$

**Figure 2.** The topology of the shortest PO's of a fully connected graph with valency  $v$  are shown together with the classical probabilities to remain, and their corresponding Lyapunov exponent.

From the previous discussion it is clear that all periodic orbits on a graph are unstable. The classical probability to remain at a specific PO of period  $t_p$  is  $M_p = \prod_{t=1}^{t_p} (M^t)_{j,j}$ . As  $M_p < 1$ , the probability to follow the PO decreases exponentially with time. Assuming regular graphs of valency  $v_j = v$  we can evaluate the rate of instability as

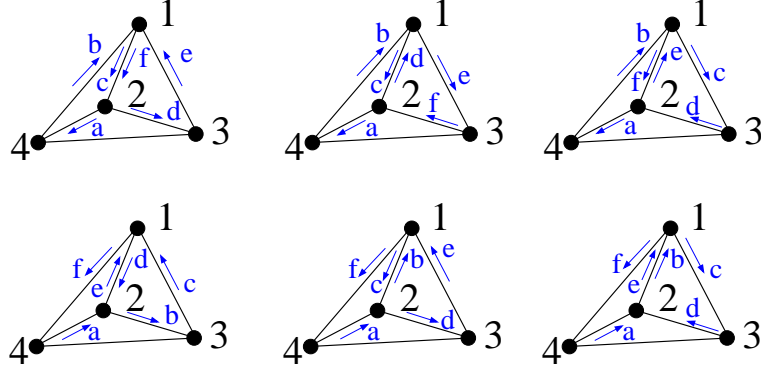
$$M_p = \prod_{s=1}^{r_p} \left(1 - \frac{2}{v}\right)^{2 t_p - r_p} \prod_{f=1}^{r_p} \left(\frac{2}{v}\right)^2 \equiv e^{-\Lambda_p t_p} \quad (13)$$

where  $\Lambda_p$  plays the role of the Lyapunov exponent (LE) and  $r_p$  is the number of vertices where back scattering occurs. For the graphs studied in this contribution, some PO's  $p$  and LE  $\Lambda_p$ , are listed in Fig. 2. The shortest PO's have period 2 and bounce back and forth between two vertices. For large graphs  $v \rightarrow \infty$  these are by far the least unstable ones, as their LE approaches 0 while all others become increasingly unstable  $\Lambda_p \sim \ln v$ .

#### 4. The spectral statistics of $U$

We consider the matrix  $U(k, A)$  defined in Eqs. (7), (8). The spectrum consists of  $2B$  points  $e^{i\epsilon_t(k)}$  confined to the unit circle (eigenphases). Unitary matrices of this type are frequently studied since they are the quantum analogues of classical, area preserving maps. Their spectral fluctuations depend on the nature of the underlying classical dynamics [31]. The quantum analogues of classically integrable maps display Poissonian statistics while in the opposite case of classically chaotic maps, the eigenphase statistics conform with the results of RMT for Dyson's *circular ensembles*. To describe the spectral fluctuations of  $U$  we consider the form factor

$$K(t, 2B) = \frac{1}{2B} \langle |\text{tr} U^t|^2 \rangle \quad (t > 0). \quad (14)$$



**Figure 3.** The family  $\mathcal{L}$  of isometric orbits  $\mathcal{F}_6$  of period  $t_p = 6$  and length  $L_p = 2l_{1,2} + l_{1,4} + l_{1,3} + l_{2,3} + l_{2,4}$  for the tetrahedron. The various orbits (6 in total) are indicated with the sequence of letters associated with the arrows.

The average  $\langle \dots \rangle$  will be specified below. RMT predicts that  $K(t, 2B)$  depends on the scaled time  $\tau = \frac{t}{2B}$  only [31], and explicit expressions for the orthogonal and the unitary circular ensembles are known [25].

Using (7), (8) we expand the matrix products in  $\text{tr} U^t$  and obtain a sum of the form

$$\text{tr} U^t(k) = \sum_{p \in \mathcal{P}_t} \mathcal{A}_p e^{i(kL_p + Al_p)}. \quad (15)$$

In this sum  $p$  runs over all closed trajectories on the graph which are compatible with the connectivity matrix and which have the topological length  $t$ , i. e. they visit exactly  $t$  vertices. For graphs, the concepts of closed trajectories and periodic orbits coincide, hence (15) can also be interpreted as a periodic-orbit sum. From (15) it is clear that  $K(t/2B) = 0$  as long as  $t$  is smaller than the period of the shortest periodic orbit. The phase associated with an orbit is determined by its total (metric) length  $L_p = \sum_{b \in p} L_b$  and by the “magnetic flux” through the orbit. The latter is given in terms of its total *directed* length  $l_p$  if we assume for simplicity that the magnitude of the magnetic vector potential is constant  $|A_b| \equiv A$ . The amplitude of the contribution from a periodic orbit by the product of all the elements of vertex-scattering matrices encountered

$$\mathcal{A}_p = \prod_{j=1}^{n_p} \sigma_{i_{j-1}, i_j}^{(i_j)} \equiv \prod_{[r,s,t]} \left( \sigma_{r,t}^{(s)} \right)^{n_p(r,s,t)}, \quad (16)$$

i. e. for fixed boundary conditions at the vertices it is completely specified by the frequencies  $n_p(r, s, t)$  of all transitions  $(r, s) \rightarrow (s, t)$ . Inserting (15) into the definition of the form factor we obtain a double sum over periodic orbits

$$K(t/2B) = \frac{1}{2B} \left\langle \sum_{p, p' \in \mathcal{P}_n} \mathcal{A}_p \mathcal{A}_{p'}^* \exp \{ik(L_p - L_{p'}) + iA(l_p - l_{p'})\} \right\rangle. \quad (17)$$

Now we have to specify our averaging procedure which has to respect the restrictions imposed by the underlying classical dynamics. To this end we will use the wavenumber



$k$  for averaging i.e.  $\langle \dots \rangle_k = \lim_{k \rightarrow \infty} k^{-1} \int_0^k dk' (\dots)$  (and, if present, also the magnetic vector potential  $A$ ). Provided that the bond lengths of the graph are rationally independent and that a sufficiently large interval is used for averaging, we have

$$\langle e^{ik(L_p - L_{p'})} \rangle_k = \delta_{L_p, L_{p'}} \quad \text{and} \quad \langle e^{iA(l_p - l_{p'})} \rangle_A = \delta_{l_p, l_{p'}} \quad (18)$$

i.e. only terms with  $L_p = L_{p'}$  and  $l_p = l_{p'}$  survive.

Note that  $L_p = L_{p'}$  does *not* necessarily imply  $p = p'$  or that  $p, p'$  are related by some symmetry because there exist families  $\mathcal{L}$  of distinct but isometric orbits which can be used to write the result of (17) in the form [1, 2, 3, 9]

$$K(t/2B) = \sum_{\mathcal{L} \in \mathcal{F}_n} \left| \sum_{p \in \mathcal{L}} \mathcal{A}_p \right|^2. \quad (19)$$

The outer sum is over the set  $\mathcal{F}_n$  of families, while the inner one is a *coherent* sum over the orbits belonging to a given family (= metric length). An example of such family for the tetrahedron is shown in Fig. 3. Eq. (19) is exact, and it represents a combinatorial problem since it does not depend any more on metric information about the graph (the bond lengths).

In general, the combinatorial problem (19) is very hard and cannot be solved in closed form. Nevertheless *exact* result for finite  $t$  can always be obtained from (19) using a computer algebra system such as Maple [32]. To this end, one has to represent  $\text{tr} U^t$  as a multivariate polynomial of degree  $t$  in the variables  $e^{ikL_i}$ , i. e.

$$\text{tr} U^t = \sum_{\mathcal{P}_t} c_{\mathcal{P}} (e^{ikL_1})^{p_1} (e^{ikL_2})^{p_2} \dots, \quad (20)$$

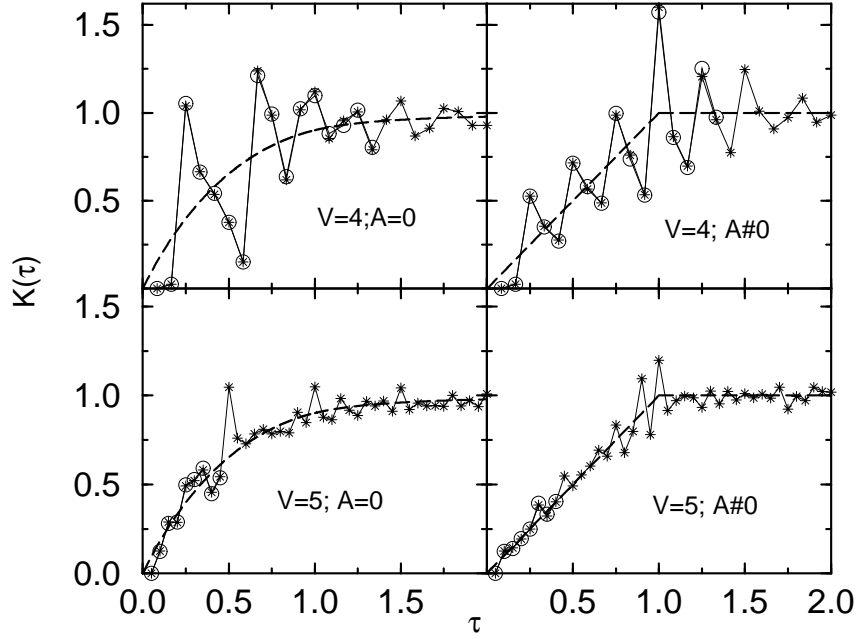
where  $\mathcal{P}_t$  runs over all partitions of  $t$  into non-negative integers  $t = p_1 + p_2 + \dots$  [9]. The form factor is then simply given as

$$K(t/2B) = \sum_{\mathcal{P}_t} |c_{\mathcal{P}}|^2. \quad (21)$$

The task of finding the coefficients  $c_{\mathcal{P}}$  can be expressed in Maple with standard functions. In Fig. 4 we compare the results of (21) with direct numerical averages for fully connected graphs with  $V = 4$  and  $V = 5$  vertices with and without magnetic field breaking the time-reversal symmetry. The results agree indeed to a high precision. Although this could be regarded merely as an additional confirmation of the numerical procedures used in [1], we see the main merit of (21) in being a very useful tool for trying to find the solution of (19) in closed form.

## 5. Wavefunction statistics

Following the quantization outlined in section 2 a quantum wavefunction is defined as a set of  $2B$  complex amplitudes  $a_d$ , normalized according to  $\sum_d |a_d|^2 = 1$ . Here we will care about stationary solution satisfying Eq. (5) (i.e. eigenstates of the graph



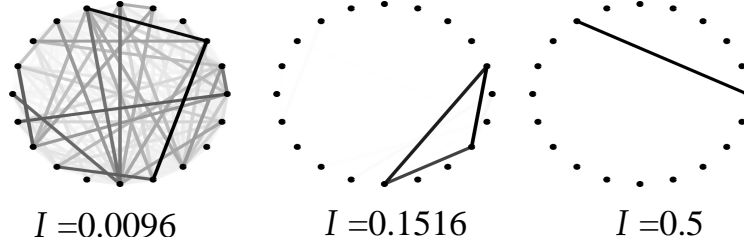
**Figure 4.** Form factor of  $U$  for regular graphs with  $V = 4$  vertices (top) and  $V = 5$  vertices (bottom). In the right panels an additional magnetic field destroyed time-reversal symmetry. Circles: exact results obtained from (19). Stars: numerical average over 2,000 values of  $k$ . The solid line is to guide the eye. The prediction of the appropriate random matrix ensemble are shown with dashed lines.

with corresponding wavelength  $k$ ). The standard localization measure is the Inverse Participation Ratio (IPR) which is defined as

$$\mathcal{I} = \sum_{d=1}^{2B} |a_d|^4. \quad (22)$$

Ergodic states which occupy each directed bond with the same probability have  $\mathcal{I} = 1/2B$  and up to a constant factor depending on the presence of symmetries this is also the RMT prediction. In the other extreme  $\mathcal{I} = 0.5$  indicates a state which is restricted to a single bond only, i. e. the greatest possible degree of localization. Some representative eigenstates are shown in Fig. 5.

The key theoretical idea discussed and applied in several recent works [33, 34, 35] is that wavefunction intensities in a complex system can often be separated into a product of short-time and long-time parts, the latter being a random variable. On the other hand the short time part can be evaluated using information about classical dynamics. Specifically we have that the probability amplitude  $A_d$  to return to the original state



**Figure 5.** Representative eigenstates for a fully connected graph with  $V = 10$ . The corresponding IPR's are (from left to right)  $\mathcal{I} = \frac{1}{104} \approx 0.0096$ ;  $\mathcal{I} = \frac{1}{6} \approx 0.1516$ ;  $\mathcal{I} = \frac{1}{2} = 0.5$

$|d\rangle$  is

$$A_d \equiv \langle d|U^t|d\rangle = \sum_m |\langle d|m\rangle|^2 e^{-i\epsilon_m t}. \quad (23)$$

The return probability is then

$$P_d(t) \equiv |A_d|^2 = \sum_{m,n} |\langle d|m\rangle|^2 |\langle d|n\rangle|^2 e^{i(\epsilon_m - \epsilon_n)t} \quad (24)$$

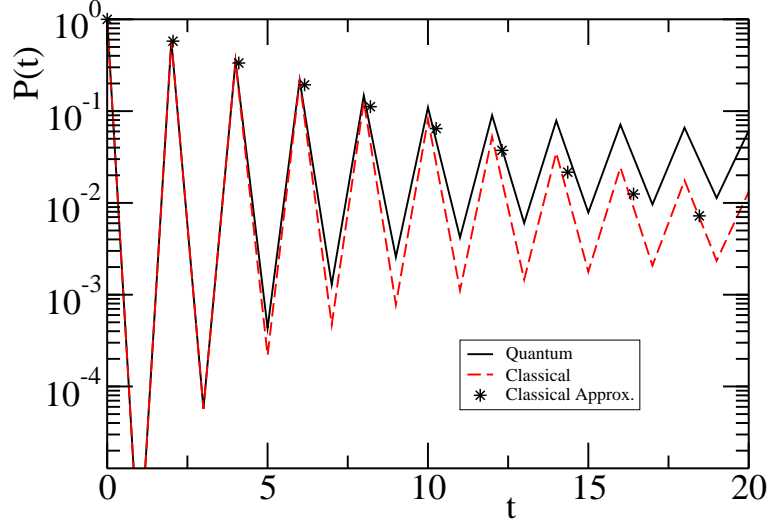
Averaging over initial states and over time (typically larger than the Heisenberg time  $t_H = 2\pi/\Delta = 2B$ ) we get

$$\overline{\langle P_d(t) \rangle}_d \equiv \langle \frac{1}{2B} \sum_{t=1}^{2B} P_d(t) \rangle_d = \frac{1}{2B} \sum_{t=1}^{2B} P(t) = \langle \sum_m |\langle d|m\rangle|^4 \rangle_d \equiv \langle \mathcal{I} \rangle_d \quad (25)$$

where  $\overline{\cdots}^t$  indicates an average over time and  $\langle \cdots \rangle_d$  over initial states. Above  $P(t)$  indicates the averaged (over initial states) return probability. In the last equality we had used the fact that due to time-average the off-diagonal terms averaged out to zero. Eq. (25) expresses the mean IPR in terms of the quantum return probability (RP), averaged over time and initial states. The next step is to argue that the quantum short-time dynamics, can be described by the classical time evolution (see Fig. 6). The latter can be approximated semiclassically quite well based only on period-two PO's which correspond to trajectories which bounce back and forth between two vertices. These type of orbits have the lowest Lyapunov exponent (LE) and it is expected to have the largest influence on eigenfunction localization because classical trajectories can cycle in their vicinity for a relatively long time and increase the RP beyond the ergodic average. The resulting survival probability is

$$P(t) = \begin{cases} 0; & t \text{ odd} \\ (-1 + \frac{2}{v})^4; & t \text{ even} \end{cases} \quad (26)$$

Indeed the period 2 orbits totally dominate the classical and quantum RP at short times as can be seen in Fig. 6. Including the contribution of these orbits only, Kaplan obtained a mean IPR which is by a factor  $\sim v$  larger than the RMT expectation, in



**Figure 6.** The quantum survival probability (solid line), the classical survival probability (dashed line) and the classical short time approximation ( $\star$ ) based only in the period-two orbits as it is given by Eq.(26).

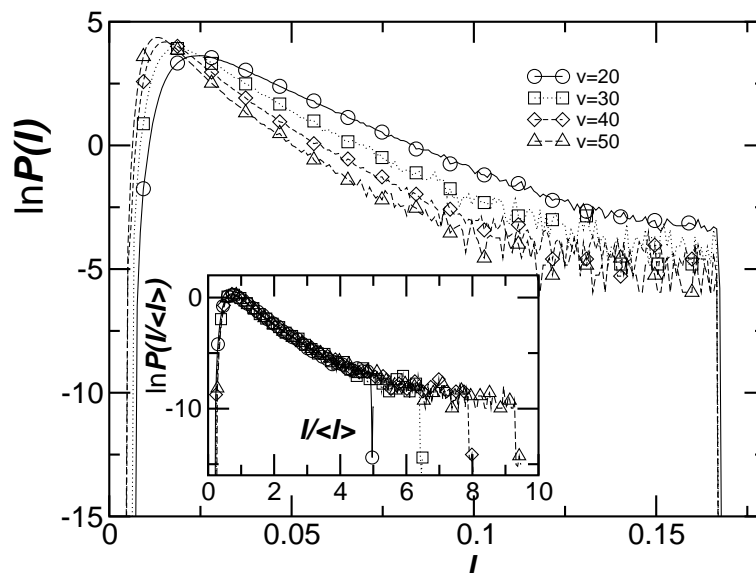
agreement with numerics [35]. Moreover, following the same line of argumentation as in [34] we get that the bulk of the IPR distribution scales as [13]

$$\tilde{P}(\mathcal{I}/\langle\mathcal{I}\rangle) = \langle\mathcal{I}\rangle P(\mathcal{I}) \quad (27)$$

indicating that the whole bulk of  $\mathcal{P}(\mathcal{I})$  is effectively determined by the least unstable orbits. This result can be nicely verified from the numerical data presented in Fig. 7.

With all this evidence for their prominent role in wavefunction localization, one clearly expects to see strong scarring on the period 2 orbits. Such states would essentially be concentrated on two directed bonds and give rise to  $\mathcal{I} \sim 1/2$ . However, in this region  $\mathcal{P}(\mathcal{I})$  is negligible (see Fig. 7). We conclude therefore that *the shortest and least unstable orbits of our system produce no visible scars*. Note that the same applies also to the value  $\mathcal{I} = 1/4$  expected from the V-shaped orbits of Fig. 2. In fact  $\mathcal{P}(\mathcal{I})$  has an appreciable value only for  $\mathcal{I} \leq 1/6$  (Fig. 7). The position of this cutoff precisely coincides with the IPR expected for states which are scarred by triangular orbits. They occupy six directed bonds since, due to time-reversal symmetry, scarring on a PO and its reversed must coincide. Indeed a closer inspection shows that the vast majority of states at  $\mathcal{I} \approx 1/6$  look like the example shown in Fig. 5. Of course the step at  $\mathcal{I} = 1/6$ , which is present for any graph size  $V$ , is incompatible with the scaling of  $P(\mathcal{I})$  mentioned above and indeed this relation breaks down in the tails at the expected points (inset of Fig. 7).

These results [13] provide clear evidence for the fact that *enhanced wavefunction localization due to the presence of short unstable orbits and strong scarring can in principle rely on completely unrelated mechanisms* and can also leave distinct traces in statistical measures such as the distribution of inverse participation ratios (IPR). As



**Figure 7.** Probability distribution  $\mathcal{P}(\mathcal{I})$  of the inverse participation numbers, showing a steplike cutoff at  $\mathcal{I} = 1/6$  that can be attributed to scarring on triangular orbits. In the inset we report the rescaled distribution  $\tilde{\mathcal{P}}(\mathcal{I}/\langle \mathcal{I} \rangle)$ . A nice scaling is observed.

a matter of fact in [13] we were able to identify a necessary and sufficient condition for the energies of perfect scars

$$(kL_d) \bmod \pi = 0 \quad \forall d \in p \quad (28)$$

where  $d$  is a directed bond which belongs to the specific PO  $p$ . Eq. (28) is reminiscent of a simple Bohr-Sommerfeld quantization condition  $kL_p = 2n\pi$ , as it applies, e. g., to strong scars in billiards. However, there is an important difference: not only does Eq. (28) require quantization of the total action  $kL_p$  of the scarred orbit, it also implies action quantization on all the visited bonds  $d$ . This stronger condition can only be met if the lengths of all bonds on  $p$  are rationally related. As in general the bond lengths are incommensurate *there are no perfect scars for generic graphs*. Nevertheless, for incommensurate bond lengths Eq. (28) can be approximated with any given precision and then visible scars are expected [13].

## 6. Conclusions and Outlook

We have reviewed some of our results on the statistical properties of eigenvalues and eigenfunctions of the unitary quantum time evolution operator derived from quantum graphs. We have concentrated on fully connected quantum graphs. For this family of graphs, the gap  $\Delta_g$  between the two maximum eigenvalues of the classical evolution operator approaches 1 as the number of directed bonds increases, thus satisfying the (sufficient) condition [16] for a graph in order to show universal spectral statistics. One

possible approach in understanding how universality emerge is the use of combinatorial methods to perform the periodic-orbit sums related to spectral two-point correlations.

At the same time, we show that the existing scar theory does not explain the appearance of visible scars (super-scars). As a matter of fact our numerical data indicated that enhanced wavefunction localization due to short unstable orbits and strong scarring are not the same thing.

Quantum graphs were proven throughout the years very useful models. They allowed us to gain a good understanding of the spectrum and eigenfunctions properties of quantum systems with underlying classical chaotic dynamics. Semiclassics on graphs is exact, and various quantum mechanical quantities can be written in terms of classical periodic orbits. These studies and their conclusions are by now well documented in the quantum chaos literature. But quantum chaology has various other challenges that wait to be addressed. Among them is a quantum mechanical theory of dynamical evolution which is still a missing chapter. Quantum dissipation, dephasing and irreversibility (also used in the framework of fidelity studies in quantum computation) of quantum chaotic motion are notions, which are related with specific aspects of this evolution. It is our believe that quantum graphs can play a prominent role in this ultimate challenge: to develop a general theory for the time evolution of quantum systems with underlying classical chaotic behavior.

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# Preparing an article for publication in an Institute of Physics Publishing journal using $\text{\LaTeX} 2_{\epsilon}$

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**Abstract.** This document describes the preparation of an article using  $\text{\LaTeX} 2_{\epsilon}$  and `iopart.cls` (the IOP  $\text{\LaTeX} 2_{\epsilon}$  preprint class file). This class file is designed to help authors produce preprints in a form suitable for submission to any of the journals published by Institute of Physics Publishing.

Authors submitting to any IOP journal, i.e. to double-column journals as well as the single-column ones, should follow the guidelines set out here. On acceptance, their source code will be converted to the appropriate journal format at Institute of Physics Publishing. For the printed version, Times fonts (and Helvetica in double-column journals) will be used instead of the Computer Modern used in the preprint form.

PACS numbers: 00.00, 20.00, 42.10

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## 1. Introduction

Institute of Physics Publishing (IOPP) publishes a wide range of research and review journals and magazines. It is wholly owned by The Institute of Physics, the United Kingdom professional body for physicists. Many of our authors use  $\text{\TeX}$  or  $\text{\LaTeX}$  to produce their typescripts and, where the source code is supplied to us, we can use it to produce the printed version; this gives more rapid publication with a smaller chance of typographical error.

This document gives the procedures and specific requirements for the preparation and presentation of text and illustrations for articles in  $\text{\LaTeX} 2_{\epsilon}$  using the Institute of Physics Publishing class file `iopart.cls`. It also illustrates the use of the class file as it has been prepared using `iopart.cls`. The class file and accompanying documentation are available to all authors and copies can be obtained

- from the World Wide Web (<http://www.iop.org/Journals/textstyle>) by downloading zipped files for PCs or tar compressed format files for Unix.
- from the World Wide Web by using our Author Enquiry Service (<http://www.iop.org/Journals/AES>) to request that the files be sent on disk.
- by e-mailing `prod2@ioppublishing.co.uk` and requesting the files be sent on  $5\frac{1}{4}$  or  $3\frac{1}{2}$  inch PC discs or on  $3\frac{1}{2}$  inch Mac disk
- by contacting the Electronic Production Assistant, Institute of Physics Publishing, Dirac House, Temple Back, Bristol BS1 6BE, UK (phone: +44 (0)117 929 7481; fax: +44 (0)117 929 4318) and requesting the files.

There are ten and twelve point versions of the style, but the twelve point version should be used for preprints and is selected by opening the file with the initial line:

```
\documentclass[12pt]{iopart}
```

If the `[12pt]` is omitted the article will be set in 10pt type (i.e. in the same size type as in the printed journal). The twelve point version of the style gives a ‘preprint’ form with a page width and type size 1.2 times larger than that for normal single-column journals with extra spacing between lines. The page depth is less than 1.2 times the normal page depth so that articles will fit on the page on both A4 and Letter paper. This form is the one required for the initial submission of a typescript for refereeing and copy editing. Authors need not aim to optimize the line and page breaks as they will inevitably change when converted to the final format for printing.

The ten point version has the same page dimensions and type sizes as a single-column journal and shows approximately how the text would appear in print. It can also be used to produce camera-ready copy for journal special issues.

Other compatible  $\text{\LaTeX} 2_{\epsilon}$  packages can be used if they are available in the normal distribution of  $\text{\LaTeX} 2_{\epsilon}$ ; if it is essential to use a non-standard package then the extra files needed to process the article must also be sent in. Authors should be aware that the final version will be printed on a different page size and using different fonts to the

preprint version so that any special effects used should not contain material that is not easily scalable.

There is also an equivalent file for Plain T<sub>E</sub>X `iopppt.tex` which is available for authors who prefer to use Plain T<sub>E</sub>X, but, although the use of one of these style files is recommended, submission of T<sub>E</sub>X files is not restricted to files using them. Articles prepared using almost any version of T<sub>E</sub>X or L<sup>A</sup>T<sub>E</sub>X can be handled (*AMST<sub>E</sub>X*, *LAMSTeX*, *PHYZZX*, etc) and authors not using the IOPP style files can submit their source code in the way described above. Alterations to the source code will be made in-house, in order to bring it in line with IOPP style.

We aim to maintain our normal standards for articles published from T<sub>E</sub>X and L<sup>A</sup>T<sub>E</sub>X files so we reserve the right to make small alterations to clarify and improve the English where necessary and to put the article into IOPP house style.

### *1.1. Double-column journals*

Authors writing for double-column journals may use the IOPP preprint macros. Conversion from the single-column format to the double-column output required for printing will be done at Institute of Physics Publishing.

## **2. Submitting typescripts and sending in files**

### *2.1. What to send on submission*

Authors are asked to prepare their articles using the 12pt version of style and submit three printed copies of the article (together with printouts of any figures) for refereeing. They should inform the journal's Managing Editor that the L<sup>A</sup>T<sub>E</sub>X source code is available, whether the graphics files are available and in what format, and how they can send the file(s), but should **not** send the file until the article has received favourable referee reports. The Managing Editor of the journal concerned will notify the author when to send the L<sup>A</sup>T<sub>E</sub>X files.

### *2.2. Sending in files*

Files may be sent by e-mail, FTP or on PC or Mac floppy disk.

*2.2.1. By e-mail.* E-mail can be used to send most files. Files should be sent to `prod2@ioppublishing.co.uk`. However, there are some points to note to ensure the files received are useable.

- long articles (above about 35 pages of typescript) should not be sent as a single file by e-mail but should either be split into a series of smaller files or sent by FTP.
- long lines may be truncated and certain ASCII characters may be corrupted. To guard against this, authors using e-mail are asked to ensure that no line in their

source code exceeds 75 characters in length, and that a simple character-check table is sent with their paper (like the one at the beginning of this file and `iopart.cls`).

- each file should be sent as a separate message; do not include more than one file in the same message.
- make sure all messages and files contain enough information for us to identify which article they relate to. Please provide the following: name of sender, first named author of article, title of article, journal submitted to (and reference number if known) and name of file. Files names should, where possible, be restricted to eight characters with three for the extension.
- files may be encrypted or compressed using standard software and included as attachments to mail messages. The message itself should describe what encryption or compression software has been used. The following programs can be used for encryption/compression: uuencode, pkzip, gzip, stuffit, MIME.

*2.2.2. By FTP.* Files can be sent by anonymous FTP to `ftp.ioppublishing.com` and should be placed in the directory `/FTP/Public/incoming/prod2`. As a security measure, files deposited in this directory are removed at regular intervals during the day. When depositing groups of files for a single paper please create a new directory within `/prod2` and then place all files within that directory. Please e-mail `prod2@ioppublishing.co.uk` to inform us that files have been deposited and to provide details of the files and the article to which they belong (i.e. journal, title, authors, reference number and list of files).

*2.2.3. On disk.* Files may be sent in on PC (3.5 inch or 5.25 inch) or Apple Macintosh disk (3.5 inch). Files should have names which indicate their contents and, for PC files, use a maximum of eight characters in the name and three in the extension. A ‘readme’ file or commented material should be included on the disk to provide information on the sender, contact address, file contents, journal submitted to, article reference number (if known) unless this information is included in a covering letter sent with the disk.

For articles which have been accepted for publication, disks should be mailed to:

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where *Journal Name* is the name of the journal to which the article has been submitted.

### 3. Preparing your article

Using L<sup>A</sup>T<sub>E</sub>X with the `iopart` class file provides a simple way of producing an article in a form suitable for publication in one of the IOPP journals. Authors may add their own macros at the start of an article provided they do not overwrite existing definitions and that they send copies of their new macros with their text file. `iopart` can be used with other package files such as those loading the AMS extension fonts `msam` and `msbm` (these fonts provide the blackboard bold alphabet and various extra maths symbols as well as symbols useful in figure captions); an extra style file `iopams.sty` is provided to load these packages and provide extra definitions for bold Greek letters.

In preparing your article you are requested to follow these guidelines as closely as possible; this will minimize the amount of copy editing required and will hasten the production process. This is particularly important with regard to the reference list.

The file name can be up to eight characters long with the suffix `.tex`. Please use file names that are likely to be unique, and include commented material to identify the journal, author and reference number if known. The first non-commented line should be `\documentclass[12pt]{iopart}` to load the preprint class file. Other standard predeclared option files can be included in square brackets; copies of any non-standard options must be sent in with the source code. Omitting `[12pt]` produces an article with the normal journal page and type sizes. Macros for the individual paper not included in a style file should be inserted in the preamble to the paper with comments to describe any complex or non-obvious ones. Authors of very long articles may find it convenient to separate their article into a series of files each containing a section, each of which is called in turn by the primary file.

The start of the article text is signalled by `\begin{document}`. The journal to which the article is to be submitted may be selected with the command `\jl{#1}`, where `#1` is the journal reference number given in table 1. This information is used if the command `\submitted` is used (see later) but is not essential to the running of the file.

### 4. The title and abstract page

The code for setting the title page information is slightly different from the normal default in L<sup>A</sup>T<sub>E</sub>X.

**Table 1.** The reference numbers for all the journals for which the IOPP macros can be used.

No	Journal
1	Journal of Physics A: Mathematical and General
2	Journal of Physics B: Atomic, Molecular and Optical Physics
3	Journal of Physics: Condensed Matter
4	Journal Physics G: Nuclear and Particle Physics
5	Inverse Problems
6	Classical and Quantum Gravity
7	Network
8	Nonlinearity
9	Quantum Optics
10	Waves in Random Media
11	Pure and Applied Optics
12	Physics in Medecine and Biology
13	Modelling and Simulation in Materials Science and Engineering
14	Plasma Physics and Controlled Fusion
15	Physiological Measurement
16	Combustion Theory and Modelling (new in 1997)
17	High Performance Polymers
18	Public Understanding of Science
19	Reports on Progress in Physics
20	Journal of Physics D: Applied Physics
21	Superconductor Science and Technology
22	Semiconductor Science and Technology
23	Nanotechnology
24	Measurement Science and Technology
25	Plasma Sources Science and Technology
26	Smart Materials Structure
27	Journal of Micromechanics and Microengineering
28	Distributed Systems Engineering
29	Bioimaging
30	Journal of Radiological Protection
31	European Journal of Physics
32	Journal of Optics

#### 4.1. Titles and article types

The title is set in bold unjustified type using the command `\title{#1}`, where **#1** is the title of the article. The first letter of the title should be capitalized with the rest in lower case. Mathematical expressions within the title may be left in light-face type rather than bold because the Computer Modern bold maths and symbol fonts may not be available at the size required for the title. The final printed version will have bold mathematical expressions in the title. If the title is unsuitable for use as a running head a short form can be provided as an optional argument (in square brackets) before the full title, i.e. `\title[Short title]{full title}`. A short title is required when the title itself is too long to be used as the short title or when the title contains a footnote.

**Table 2.** Types of article defined in the `iopart.cls` class file.

Command	Type	Heading on first page
<code>\title{#1}</code>	Paper	—
<code>\review{#1}</code>	Review	REVIEW
<code>\topical{#1}</code>	Topical review	TOPICAL REVIEW
<code>\comment{#1}</code>	Comment	COMMENT
<code>\note{#1}</code>	Note	NOTE
<code>\paper{#1}</code>	Paper	—
<code>\prelim{#1}</code>	Preliminary communication	PRELIMINARY COMMUNICATION
<code>\rapid{#1}</code>	Rapid communication	RAPID COMMUNICATION
<code>\letter{#1}</code>	Letter	LETTER TO THE EDITOR
<code>\article{#1}{#2}</code>	Other articles	Whatever is entered as #1

For articles other than papers the IOPP class file, `iopart.cls`, allows a generic heading `\article[Short title]{TYPE}{Full title}` and the specific definitions given in table 2. In each case (apart from Letters to the Editor) an optional argument can be used immediately after the control sequence name to specify the short title; where no short title is given the full title will be used as the running head at the top of each page apart from the first. For Letters no short title is required as the running head is automatically defined to be *Letter to the Editor*. The generic heading could be used for articles such as those presented at a conference or workshop, e.g.

`\article[Short title]{WORKSHOP ON HIGH-ENERGY PHYSICS}{Title}`

#### 4.2. Authors' names and addresses

The next information required is the list of authors' names and their affiliations. For the authors' names type `\author{#1}`, where #1 is the list of all authors' names. The style for the names is initials then surname, with a comma after all but the last two names, which are separated by 'and'. Initials should *not* have full stops. Christian names may be used if desired. If the authors are at different addresses one of the symbols †, ‡, §, ||, ¶, +, \*, ‡ should be used after each surname to reference an author to his/her address. The symbols should be used in the order given. If an author has additional information to appear as a footnote, such as a permanent address, and the footnote symbols are not being used to identify an address, the footnote should be entered after the surname as a normal L<sup>A</sup>T<sub>E</sub>X footnote, without specifying a sign. Where footnote symbols are being used to indicate which address the author is at, the symbol used for a footnote should be the next one from the list given above and has to be selected individually using the command `\footnote[<num>]{Text of footnote}`, where <num> is a number representing the position of the desired symbol in the list above, i.e. for 1 for †, 2 for ‡, etc.

The addresses of the authors' affiliations follow the list of authors. Each address is set by using `\address{#1}` with the address as the single parameter in braces. If there is more than one address then the appropriate symbol should come at the start of the address.

#### 4.3. The abstract

The abstract follows the addresses and should give readers concise information about the content of the article and indicate the main results obtained and conclusions drawn. It should be complete in itself with no table numbers, figure numbers or references included and should not normally exceed 200 words. To indicate the start of the abstract type `\begin{abstract}` followed by the text of the abstract (not in braces). The abstract should normally be restricted to a single paragraph and is terminated by the command `\end{abstract}`

#### 4.4. Subject classification numbers

Following the abstract come any Physics and Astronomy Classification System (PACS) codes or American Mathematical Society (AMS) classification scheme numbers. The command `\pacs{#1}`, with the subject classification numbers from the Physics and Astronomy Classification Scheme as the parameter, defines the subject area of the paper (or for a single number `\pacno{#1}`). If PACS numbers are not readily available, *Physics Abstracts* classification scheme numbers can be given instead. If this command is omitted the classification numbers for indexing will be allocated by IOPP staff. It is unnecessary to supply PACS numbers for *Inverse Problems* and *Physics in Medicine and Biology*. AMS classification numbers may be given as well as, or instead of, PACS numbers for mathematical articles, they are specified using the command `\ams{#1}`.

After any classification numbers the command `\submitted` can be inserted to print out a line indicating the article has been submitted to the appropriate Institute of Physics Journal. This command is optional.

#### 4.5. Making a separate title page

The command `\maketitle` forces a page break after the point where it is inserted and so to keep the header material on a separate page from the body of the text insert `\maketitle` or `\newpage` after the classification codes and `\submitted` commands (if present) or the end of the abstract. If `\maketitle` is not included the text of the article will start immediately after the abstract.

#### 4.6. Sample coding for the start of an article

The code for the start of a title page of a typical paper might read:

```
\documentclass[12pt]{iopart}
```

```
\begin{document}
\jl{4}
\title[The anomalous magnetic moment of the
neutrino]{The anomalous magnetic moment of the
neutrino and its relation to the solar neutrino problem}

\author{P J Smith\dag, T M Collins\ddag,
R J Jones\ddag\footnote[3]{Present address:
Department of Physics, University of Bristol, Tyndalls Park Road,
Bristol BS8 1TS, UK.} and Janet Williams\P}

\address{\dag\ Mathematics Faculty, Open University,
Milton Keynes MK7~6AA, UK}
\address{\ddag\ Department of Mathematics,
Imperial College, Prince Consort Road, London SW7~2BZ, UK}
\address{\P\ Department of Computer Science,
University College London, Gower Street, London WC1E~6BT, UK}

\begin{abstract}
...
\end{abstract}

\pacs{1315, 9440T}
\submitted
\maketitle
```

## 5. The text

### 5.1. Sections, subsections and subsubsections

The text of papers and reviews, but not comments or letters, should be divided into sections, subsections and, where necessary, subsubsections. To start a new section, end the previous paragraph and then include `\section` followed by the section heading within braces. Numbering of sections is done *automatically* in the headings: sections will be numbered 1, 2, 3, etc, subsections will be numbered 2.1, 2.2, 3.1, etc, and subsubsections will be numbered 2.3.1, 2.3.2, etc. Cross references to other sections in the text should, where possible, be made using labels (see section 8) but can also be made manually. Subsections and subsubsections are similar to sections but the commands are `\subsection` and `\subsubsection` respectively. Sections have a bold heading, subsections an italic heading and subsubsections an italic heading with the text following on directly.

```
\section{This is the section title}
```



`\subsection{This is the subsection title}`

The first section is normally an introduction, which should state clearly the object of the work, its scope and the main advances reported, with brief references to relevant results by other workers. In long papers it is helpful to indicate the way in which the paper is arranged and the results presented.

For articles not divided into sections, precede the start of the text (without leaving a blank line) with the command `\nosections`, which provides the appropriate space and causes the paragraph indentation to be cancelled for the first paragraph.

Footnotes should be avoided whenever possible. If required they should be used only for brief notes that do not fit conveniently into the text. The standard L<sup>A</sup>T<sub>E</sub>X macro `\footnote` should be used and will normally give an appropriate symbol; if a footnote sign needs to be specified directly `\footnote[<num>]{Text}` can be used instead where `<num>` is the number of the appropriate symbol as discussed in relation to authors (1 = `\dag`, 2 = `\ddag`, 3 = `\S`, etc).

## 5.2. Appendices

Technical detail that it is necessary to include, but that interrupts the flow of the article, may be consigned to an appendix. If there are two or more appendices they will be called Appendix A, Appendix B, etc. Numbered equations will be in the form (A.1), (A.2), etc, figures will appear as figure A1, figure B1, etc and tables as table A1, table B1, etc.

The command `\appendix` is used to signify the start of the appendixes. Thereafter `\section`, `\subsection`, etc, will give headings appropriate for an appendix. To obtain a simple heading of ‘Appendix’ use the code `\section*{Appendix}`. If it contains numbered equations, figures or tables the command `\appendix` should precede it and `\setcounter{section}{1}` must follow it.

## 5.3. Acknowledgments

Authors wishing to acknowledge assistance or encouragement from colleagues, special work by technical staff or financial support from organizations should do so in an unnumbered Acknowledgments section immediately following the last numbered section of the paper. The command `\ack` sets the acknowledgments heading as an unnumbered section. For Letters `\ack` does not set a heading but leaves a line space and does not indent the next paragraph.

## 5.4. Some matters of style

It will help your readers if your article is written in a clear, consistent and concise manner. Copy preparation at Institute of Physics Publishing will try to make sure that your work is presented to its readers in the best possible way without sacrificing the individuality of your writing.

The main elements of consistency and style we look for are presented in the booklet *Notes for Authors* (available upon request from Institute of Physics Publishing, Dirac House, Temple Back, Bristol, BS1 6BE, UK or from our World Wide Web site (<http://www.iop.org/Journals/nfa/>)). Some recommended points to note, however, are the following.

- (i) Authors are often inconsistent in the use of ‘ize’ and ‘ise’ endings. We recommend using ‘-ize’ spellings (diagonalize, renormalization, minimization, etc) but there are some common exceptions to this, for example: devise, promise and advise.
- (ii) English spellings are preferred (colour, flavour, behaviour, tunnelling, artefact, focused, focusing, fibre, etc). We write of a computer program on disk; otherwise, we use ‘programme’ and ‘disc’.
- (iii) Compound words beginning ‘non-’ or ‘self-’ are easier to read and understand when hyphenated (non-existent, self-consistent, etc).
- (iv) The words table, figure, equation and reference should be written in full and **not** contracted to Tab., fig., eq. and ref.

It will help the copy preparation and avoid unnecessary errors if you carefully check your article for accuracy, consistency and clarity before submission. Remember that your article will probably be read by many people whose native language is not English and who may not therefore be aware of many of the subtle meanings of words or idiomatic phrases present in the English language. It therefore helps if you try and keep sentences as short and simple as possible.

## 6. Mathematics

### 6.1. Two-line constructions

The great advantage of T<sub>E</sub>X and L<sup>A</sup>T<sub>E</sub>X over other text processing systems is their ability to handle mathematics to almost any degree of complexity. However, in order to produce an article suitable for publication within a journal, authors should exercise some restraint on the constructions used. For simple fractions in the text the solidus /, as in  $\lambda/2\pi$ , should be used instead of `\frac` or `\over`, care being taken to use parentheses where necessary to avoid ambiguity, for example to distinguish between  $1/(n-1)$  and  $1/n-1$ . Exceptions to this are the proper fractions  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{3}{4}$ , etc, which are better left in this form. In displayed equations horizontal lines are preferable to solidi provided the equation is kept within a height of two lines. A two-line solidus should be avoided where possible; the construction  $(\dots)^{-1}$  should be used instead; for example use:

$$\frac{1}{M_a} \left( \int_0^\infty d\omega \frac{|S_o|^2}{N} \right)^{-1}$$

instead of

$$\frac{1}{M_a} / \int_0^\infty d\omega \frac{|S_o|^2}{N}.$$

## 6.2. Roman and italic in mathematics

In mathematics mode L<sup>A</sup>T<sub>E</sub>X automatically sets variables in an italic font. In most cases authors should accept this italicization. However, there are some cases where it is better to use a Roman font; for instance, IOPP journals use a Roman d for a differential d, a Roman e for an exponential e and a Roman i for the square root of  $-1$ . To accommodate this and to simplify the typing of equations we have provided some extra definitions. `\rmd`, `\rme` and `\rmi` now gives Roman d, e and i respectively for use in equations, e.g.  $\int x e^{2x} dx/dy$  is obtained by typing `\$ \rmi x \rme^{\{2x\}} \rmd x / \rmd y \$`.

Certain other common mathematical functions, such as cos, sin, det and ker, should appear in Roman type. L<sup>A</sup>T<sub>E</sub>X provides macros for most of these functions (in the cases above, `\cos`, `\sin`, `\det` and `\ker` respectively), we have also provided additional definitions for Tr, tr and O (`\Tr`, `\tr` and `\Or`, respectively).

Subscripts and superscripts should be in Roman type if they are labels rather than variables or characters that take values. For example in the equation

$$\epsilon_m = -g\mu_n Bm$$

$m$ , the  $z$  component of the nuclear spin, is italic because it can have different values whereas  $n$  is Roman because it is a label meaning nuclear ( $\mu_n$  is the nuclear magneton).

## 6.3. Alignment of mathematics

**6.3.1. Alignment on the secondary margin.** IOPP style for displayed mathematics in single-column journals is not to centre equations, as L<sup>A</sup>T<sub>E</sub>X normally does, but to have each equation indented to a secondary margin a fixed distance from the left-hand margin of the text, except for long equations that will just fit on one line, or need to be continued on subsequent lines, which start full left. Any continuation lines are indented the fixed amount. The macros in the IOPP preprint style automatically line equations up on the secondary margin unless they are set within double dollar signs. Thus the use of double dollar signs should be avoided and the alternative `\[ ... \]` should be used instead for unnumbered equations. The equation environment should normally be used for numbered single-line equations and the `eqnarray` array environment for multiline equations. It is then only necessary to indicate which lines should start full left and this is done by including `\fl` (full left) at the start of the lines. Thus the equations:

$$\phi_k(\vec{r}) = (2\pi)^{-3/2} \exp(i\vec{k} \cdot \vec{r}) \quad (1)$$

$$N^+ = \exp(\frac{1}{2}\pi\nu)\Gamma(1 - i\nu). \quad (2)$$

are set with the code

```
\begin{eqnarray}
\phi_{\vec{k}}(\vec{r})=(2\pi)^{-3/2} \exp(\rmi\vec{k}\cdot\vec{r}) \ \ \
N^+=\exp(\case12\pi\nu)\Gamma(1-\rmi\nu) .
\end{eqnarray}
```

Where an equation will not fit on a line if indented but would if it were not, then the equation is started full left and this is achieved simply by adding `\fl` to the start of the line. For example the equation

$$R_{\rho lm, \rho' l' m'}(E) = \frac{1}{r_0} \sum_{i,j} \langle \rho l m r_0 | \Phi_i \rangle [(H_\Omega + B) - ES_\Omega]_{i,j}^{-1} \langle \Phi_j | \rho' l' m' r_0 \rangle. \quad (3)$$

does not fit on the line if indented to the secondary margin but fits in comfortably when full left.

For equations which do not fit on one line, even if started full left, the first line should be set full left with the turnover lines at the secondary margin. This is achieved by using the `eqnarray` environment and adding `\fl` at the start of the first line and `\l` at the end of each line (apart from the final line of the equation). Equations should be split at mathematically sound points, often at  $=$ ,  $+$  or  $-$  signs or between terms multiplied together. The connecting signs are not repeated and appear only at the beginning of the turned-over line. A multiplication sign should be added to the start of turned-over lines where the break is between two multiplied terms. Where an equation is broken at an equals sign (or similar, i.e.  $\equiv$ ,  $\leq$ ,  $\sim$ , etc) the sign is made more prominent by aligning it to the left of the secondary margin; where it is a  $+$ ,  $-$  or  $\times$  the sign goes to the right. Alignment to the left of the secondary margin is achieved by adding `\lo` in front of the sign (and enclosing the sign within braces if it consists of more than one character or control sequence, e.g. `\lo{:=}`). An example demonstrating these features is:

$$\begin{aligned} \langle \cos(q\Omega_s) \rangle &= \frac{1}{2} \int_0^\infty \frac{k_s(b)}{k_s^{\text{tot}}} \{ \cos[q\Omega_S^o(b_f, R_x^s)] + \cos[q\Omega_S^i(b_f, R_x^s)] \} 2\pi b db \\ &= \sum_c \frac{(\mu_c^s)^2 / |\Delta V'_s(R_c^s)|}{\sum_n (R_n^s \mu_n^s)^2 (1 - V_n^s/E)^{1/2} / |\Delta'_s(R_n^s)|} \\ &\quad \times \frac{1}{2} \int_0^{b_{\text{max}}} \{ \cos[q\Omega_S^o(b_f, R_c^s)] + \cos[q\Omega_S^i(b_f, R_c^s)] \} b db / v_s(b, R_c^s). \end{aligned} \quad (4)$$

where a simplified version of the code used is:

```
\begin{eqnarray}
\fl    <first line>  \nonumber\\
\lo=   <second line> \nonumber\\
\times <third line>
\end{eqnarray}
```

Note that alignment at the secondary margin normally takes precedence over aligning equals signs so there is usually no need to include any ampersands within the `eqnarray` environment.

*6.3.2. Secondary alignment.* While the primary alignment either on the secondary margin or full left is adequate in most cases there are examples where additional alignment is desirable. Firstly, for repeated series of short equations, secondly for equations with attached conditions and thirdly for connected series of equations with

a short left-hand side which together occupy more than a full line but where each individual part is short. In these cases the `eqnarray` environment should be used; there will still be alignment at the secondary margin but ampersands should be positioned to provide the secondary alignment. For equations with conditions the space separating the longest part from its condition is provided by `\qqquad`. Examples of equations requiring secondary alignment are:

$$A^{(3/2)} = A^{(+)} - A^{(-)} \quad (I = \tfrac{3}{2}) \quad (5)$$

$$A^{(1/2)} = A^{(+)} + 2A^{(-)} \quad (I = \tfrac{1}{2}) \quad (6)$$

$$A^{(0)} \quad (\text{otherwise}). \quad (7)$$

which is obtained with the code

```
\begin{eqnarray}
A^{(3/2)}=A^{(+)}-A^{(-)}\&(I=\case32)\&\\
A^{(1/2)}=A^{(+)}+2A^{(-)}\&\qqquad\&(I=\case12)\&\\
A^{(0)}\&(\rm otherwise).\&\\
\end{eqnarray}
```

and

$$C(12) = [\vec{\pi}(x) \cdot \vec{\phi}(x+r)]$$

$$\simeq 1 - \text{const} \frac{r^2}{L^2} \int_r^L \frac{x \, dx}{x^2} + \dots \quad (8)$$

$$\simeq 1 - \text{const} \frac{r^2}{L^2} \ln \left( \frac{L}{r} \right) + \dots \quad (9)$$

for which the code is

```
\begin{eqnarray}
C(12)\&=[\vec{\pi}(x)\cdot\vec{\phi}(x+r)]\nonumber\&\\
\&\simeq 1-\{\rm const\}\{r^2\over L^2\}\int^L_r\{x\over x^2\}+\cdots\&\\
\&\simeq 1-\{\rm const\}\{r^2\over L^2\}\ln\left(\{L\over r\}\right)+\cdots.\&\\
\end{eqnarray}
```

#### 6.4. Displayed equations in double-column journals

The way equations are displayed in the Institute of Physics Publishing double-column journals differs from that in single-column journals. However authors submitting to double-column journals can produce their equations as described above for single-column journals and conversion to the proper double-column format will take place here as proofs are being prepared.

#### 6.5. Special characters for mathematics

Bold italic characters are used in our journals to signify vectors (rather than using an upright bold or an over arrow). To obtain this effect use `\bi{#1}` within maths mode,

e.g. ***ABCdef***. If upright bold characters are required in maths use `\mathbf{#1}` within maths mode, e.g. ***XYZabc***. The calligraphic (script) uppercase alphabet is obtained with `\mathcal{AB}` or `\cal{CD}` (*ABCD*).

The American Mathematical Society provides a series of extra symbol fonts to use with L<sup>A</sup>T<sub>E</sub>X and packages containing the character definitions to use these fonts. Authors wishing to use Fraktur or Blackboard Bold can include the appropriate AMS package (e.g. `amsgen`, `amsfonts`, `amsbsy`, `amssymb`) with a `\usepackage` command or add the command `\usepackage{iopams}` which loads the four AMS packages mentioned above and also provides definitions for extra bold characters (all Greek letters and some additional other symbols).

The package `iopams` uses the definition `\boldsymbol` in `amsbsy` which allows individual non-alphabetical symbols and Greek letters to be made bold within equations. The bold Greek lowercase letters are obtained with the commands `\balpha` ... `\bomega` (but note that bold eta is `\bfeta` rather than `\beta`) and the capitals with commands `\bGamma` ... `\bOmega`. Bold versions of the following symbols are predefined in `iopams`: bold partial `\bpartial`, bold ‘ell’ `\bell`, bold  $\imath$  `\bimath`, bold  $\jmath$  `\bjmath`, bold infinity `\binfty`, bold nabla `\bnabla`, bold centred dot `\bdot`, other characters are made bold using `\boldsymbol{\symbolname}`.

Table 3 lists some other macros for use in mathematics with a brief description of their purpose. Both `\ms` (medium space) and `\bs` (big space) can be used to provide extra spacing between lines of a displayed equation or table. This space may be necessary when several separate equations are within the same equation environment.

## 6.6. Miscellaneous points

Exponential expressions, especially those containing subscripts or superscripts, are clearer if the notation `exp(...)` is used except for simple examples. For instance `exp[i(kx - ωt)]` and `exp(z2)` are preferred to `ei(kx-ωt)` and `ez2`, but `e2` is acceptable. Similarly the square root sign  $\sqrt{\phantom{x}}$  should only be used with relatively simple expressions, e.g.  $\sqrt{2}$  and  $\sqrt{a^2 + b^2}$ ; in other cases the power  $1/2$  should be used.

It is important to distinguish between  $\ln = \log_e$  and  $\lg = \log_{10}$ . Braces, brackets and parentheses should be used in the following order: `{[( )]}`. The same ordering of brackets should be used within each size. However, this ordering can be ignored if the brackets have a special meaning (e.g. if they denote an average or a function). Decimal fractions should always be preceded by a zero: for example 0.123 **not** .123. For long numbers commas are not inserted but instead a thin space is added after every third character away from the position of the decimal point unless this leaves a single separated character: e.g. 60 000, 0.123 456 78 but 4321 and 0.7325.

Equations that are referred to in the text should be numbered with the number on the right-hand side.

**Table 3.** Other macros defined in IOPP macros for use in maths.

Macro	Result	Description
Spaces		
<code>\fl</code>		Start line of equation full left
<code>\ms</code>		Spread out lines in displayed equations slightly ( $\sim 3\text{pt}$ )
<code>\bs</code>		Bigger space ( $\sim 6\text{pt}$ ) to separate lines in displays
<code>\ns</code>		Small negative space between lines in displays
For symbols to left of 5 pica indent		
<code>\lo{#1}</code>	$\#1$	Any symbol overhanging to left
<code>\eq1</code>	$=$	Left overhanging equals sign
<code>\lsim</code>	$\sim$	Left overhanging tilde
<code>\lsimeq</code>	$\simeq$	Left overhanging approximately equals
<code>\lequiv</code>	$\equiv$	Left overhanging equivalent sign
Miscellaneous		
<code>\case{#1}{#2}</code>	$\frac{\#1}{\#2}$	Text style fraction in display
<code>\Tr</code>	Tr	Roman Tr (Trace)
<code>\tr</code>	tr	Roman tr (trace)
<code>\Or</code>	O	Roman O (of order of)
<code>\tdot{#1}</code>	$\ddot{x}$	Triple dot over character
<code>\lshad</code>	$\llbracket$	Text size left shadow bracket
<code>\rshad</code>	$\rrbracket$	Text size right shadow bracket

### 6.7. Equation numbering

L<sup>A</sup>T<sub>E</sub>X provides facilities for automatically numbering equations and these should be used where possible. Sequential numbering (1), (2), etc, is the default numbering system although, if the command `\eqnobysec` is included in the preamble, equation numbering by section is obtained, e.g. (2.1), (2.2), etc. In articles with several appendixes equation numbering by section is useful in the appendixes even when sequential numbering has been used throughout the main body of the text and is switched on by the `\appendix` command. Equation numbering by section *must* be used for *Reports on Progress in Physics*. When referring to an equation in the text, either put the equation number, in brackets, e.g. ‘as in (2)’, or spell out the word equation in full, e.g. ‘if equation (2) is factorized’; do not use abbreviations such as eqn or eq. When cross-referencing is used, `\ref{<label>}` will produce ‘(<eqnum>)', `\eref{<label>}` produces ‘equation (<eqnum>)' and `Eref{<label>}` produces ‘Equation (<eqnum>)', where <label> is the label to produce equation number <eqnum>.

If an equation number is centred between lines then the command `\eqalign{...}` can be used within the ‘equation’ environment. After `\begin{equation}` enclose the lines over which the number is to be centred within `\eqalign{...}` with `\l` or `\cr` at

the end of each line. Ampersands are unnecessary within the `\eqalign` but can be used for secondary alignment if necessary. The code

```
\begin{equation}
\eqalign{T_{11}&=(1+P_e)I_{\uparrow\uparrow}-(1-P_e)
I_{\uparrow\downarrow}\\
T_{-1-1}&=(1+P_e)I_{\downarrow\downarrow}-(1-P_e)
I_{\uparrow\downarrow}\\
S_{11}&=(3+P_e)I_{\downarrow\uparrow}-(3-P_e)I_{\uparrow\uparrow}\\
S_{-1-1}&=(3+P_e)I_{\uparrow\downarrow}-(3-P_e)
I_{\downarrow\downarrow}}
\end{equation}
```

gives four equations with a centred number:

$$\begin{aligned}
 T_{11} &= (1 + P_e)I_{\uparrow\uparrow} - (1 - P_e)I_{\uparrow\downarrow} \\
 T_{-1-1} &= (1 + P_e)I_{\downarrow\downarrow} - (1 - P_e)I_{\uparrow\downarrow} \\
 S_{11} &= (3 + P_e)I_{\downarrow\uparrow} - (3 - P_e)I_{\uparrow\uparrow} \\
 S_{-1-1} &= (3 + P_e)I_{\uparrow\downarrow} - (3 - P_e)I_{\downarrow\downarrow}
 \end{aligned} \tag{10}$$

Note that the secondary alignment at the equals signs would not normally be necessary but is included here for demonstration purposes.

Sometimes it is useful to number equations as parts of the same basic equation. This can be accomplished by inserting the commands `\numparts` before the equations concerned and `\endnumparts` when reverting to the normal sequential numbering. The equations below show the previous equations numbered as separate parts using `\numparts ... \endnumparts` and the `eqnarray` environment

$$T_{11} = (1 + P_e)I_{\uparrow\uparrow} - (1 - P_e)I_{\uparrow\downarrow} \tag{11a}$$

$$T_{-1-1} = (1 + P_e)I_{\downarrow\downarrow} - (1 - P_e)I_{\uparrow\downarrow} \tag{11b}$$

$$S_{11} = (3 + P_e)I_{\downarrow\uparrow} - (3 - P_e)I_{\uparrow\uparrow} \tag{11c}$$

$$S_{-1-1} = (3 + P_e)I_{\uparrow\downarrow} - (3 - P_e)I_{\downarrow\downarrow} \tag{11d}$$

### 6.8. Miscellaneous extra commands for displayed equations

The `\cases` command of Plain  $\text{\TeX}$  is available for use with  $\text{\LaTeX}$  but has been amended slightly to increase the space between the equation and the condition. Equation (12) demonstrates simply the output from the `\cases` command

$$X = \begin{cases} 1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases} \tag{12}$$

The code used was:

```
\begin{equation}
\label{cases}
X=\cases{1&for $x \ge 0$\\
```



```
-1&for $x<0$\}
\end{equation}
```

To obtain text style fractions within displayed maths the command `\case{#1}{#2}` can be used (see equations (2) and (5)) instead of the usual `\frac{#1}{#2}` command or `{#1 \over #2}`.

When two or more short equations are on the same line they should be separated by a ‘quad space’ (`\quad`), rather than `\quad` or any combination of `\,`, `\>`, `\;` and `\ .`

## 7. Referencing

Two different styles of referencing are in common use: the Harvard alphabetical system and the Vancouver numerical system. All the IOPP journals allow the use of the Harvard system but the numerical system should **not** be used in *Physics in Medicine and Biology*. Brief descriptions of the use of the two referencing systems are given below.

### 7.1. Harvard system

In the Harvard system the name of the author appears in the text together with the year of publication. As appropriate, either the date or the name and date are included within parentheses. Where there are only two authors both names should be given in the text; if there are more than two authors only the first name should appear followed by ‘*et al*’ (which can be obtained by typing `\etal`). When two or more references to work by one author or group of authors occur for the same year they should be identified by including a, b, etc after the date (e.g. 1986a). If several references to different pages of the same article occur the appropriate page number may be given in the text, e.g. Kitchen (1982, p 39).

The reference list at the end of an article consists of an unnumbered section containing an alphabetical listing by authors’ names and in date order for each author or group of identical authors. The reference list in the preprint style is started by including the command `\section*{References}` and then `\begin{harvard}`. There will be two basic types of entries within the reference list: (i) those to journal articles and (ii) those to books, conference proceedings and reports. For both of these types of references `\item[]` is required before the start of an individual reference. The reference list is completed with `\end{harvard}`. There is also a shortened form of the coding; `\section*{References}` and `\begin{harvard}` can be replaced by the single command `\References` and `\end{harvard}` can be shortened to `\endrefs`.

*7.1.1. References to journal articles.* A normal reference to a journal article contains three changes of font: the authors and date appear in Roman type, the journal title in italic, the volume number in bold and the page numbers in Roman again. A typical journal entry would be:

Cisneros A 1971 *Astrophys. Space Sci.* **10** 87

which would be obtained by typing, within the references environment

```
\item[] Cisneros A 1971 {\it Astrophys. Space Sci.} {\bf 10} 87
```

Features to note are the following.

(i) The authors should be in the form surname (with only the first letter capitalized) **followed** by the initials with **no** periods after the initials. Authors should be separated by a comma except for the last two which should be separated by ‘and’ with no comma preceding it. For journals that accept titles of articles in the reference list, the title should be in Roman (upright) lower case letters, except for an initial capital, and should follow the date.

(ii) The journal is in italic and is abbreviated. Appendix B gives a list of macros that will give the correct abbreviation for many of the common journals. If a journal has several parts denoted by different letters the part letter should be inserted after the journal in Roman type, e.g. *Phys. Rev. A*. An exception to this is *Physics Letters* where the part letter is included in the volume number.

(iii) The volume number is bold; the page number is Roman. Both the initial and final page numbers should be given where possible. The final page number should be in the shortest possible form and separated from the initial page number by an en rule (–), e.g. 1203–14.

(iv) Where there are two or more references with identical authors, the authors’ names should not be repeated but should be replaced by \dash on the second and following occasions. Thus

```
\item[] Davis R, Mann A K and Wolfenstein L 1989 {\it Ann. Rev. Nucl.
Part. Sci.} {\bf 39} 467
\item[] \dash 1990 Private communication
```

*7.1.2. References to books, conference proceedings and reports.* References to books, proceedings and reports are similar, but have only two changes of font. The authors and date of publication are in Roman, the title of the book is in italic, and the editors, publisher, town of publication and page number are in Roman. A typical reference to a book and a conference paper might be

Dorman L I 1975 *Variations of Galactic Cosmic Rays* (Moscow: Moscow State University Press) p 103  
 Caplar R and Kulisic P 1973 *Proc. Int. Conf. on Nuclear Physics (Munich)* vol 1 (Amsterdam: North-Holland/American Elsevier) p 517

which would be obtained by typing

```
\item[] Dorman L I 1975 {\it Variations of Galactic Cosmic Rays}
(Moscow: Moscow State University Press) p~103
\item[] Caplar R and Kulisic P 1973 {\it Proc. Int. Conf. on Nuclear
Physics (Munich)} vol~1 (Amsterdam: North-Holland/American
Elsevier) p~517
```

respectively.

Features to note are the following.

(i) Book titles are in *italic* and should be spelt out in full with initial capital letters for all except minor words. Words such as Proceedings, Symposium, International, Conference, Second, etc should be abbreviated to Proc., Symp., Int., Conf., 2nd, respectively, but the rest of the title should be given in full, followed by the date of the conference and the town or city where the conference was held. For Laboratory Reports the Laboratory should be spelt out wherever possible, e.g. *Argonne National Laboratory Report*.

(ii) The volume number as, for example, vol 2, should be followed by the editors, if any, in a form such as ed A J Smith and P R Jones. Use *et al* if there are more than two editors. Next comes the town of publication and publisher, within brackets and separated by a colon, and finally the page numbers preceded by p if only one number is given or pp if both the initial and final numbers are given.

Cross referencing between the text and the reference list is not necessary for alphabetic referencing in the Harvard system as adding or deleting a reference does not normally change any of the other references.

## 7.2. Numerical system

In the numerical system references are numbered sequentially throughout the text. The numbers occur within square brackets and one number can be used to designate several references. A numerical reference list in the preprint style is started by including the command `\section*{References}` and then `\begin{thebibliography}{<num>}`, where `<num>` is the largest number in the reference list (or any other number with the same number of digits). The reference list gives the references in numerical, not alphabetical, order and is completed by `\end{thebibliography}`. Short forms of the commands are again available: `\Bibliography{<num>}` can be used at the start of the references section and `\endbib` at the end. (Note that footnotes should not be part of a numerical reference system, but should be included in the text using the symbols †, ‡, etc.)

References to journals and books are similar to those in the Harvard system, except that two or more references with identical authors are spelt out in full, i.e. they are **not** replaced with `\dash`. When one number covers two or more separate references `\nonum` or `\par\item[]` should be included at the start of each reference in a group after the first. A typical numerical reference list might begin

- [1] Dorman L I 1975 *Variations of Galactic Cosmic Rays* (Moscow: Moscow State University Press) p 103
- [2] Caplar R and Kulisic P 1973 *Proc. Int. Conf. on Nuclear Physics (Munich)* vol 1 (Amsterdam: North-Holland/American Elsevier) p 517
- [3] Cisneros A 1971 *Astrophys. Space Sci.* **10** 87

which would be obtained by typing

```
\item Dorman L I 1975 {\it Variations of Galactic Cosmic Rays}
```

```
(Moscow: Moscow State University Press) p~103
\item Caplar R and Kulisic P 1973 {\it Proc. Int. Conf.
on Nuclear Physics (Munich)} vol~1 (Amsterdam:
North-Holland/American Elsevier) p~517
\item Cisneros A 1971 {\it Astrophys. Space Sci.} {\bf 10} 87
```

The point to note is that this is identical to the entries in the Harvard system except that square brackets following `\item` are no longer required.

### 7.3. Reference lists

A complete reference should provide the reader with enough information to locate the article concerned and should consist of: name(s) and initials, date published, title of journal or book, volume number, editors, if any, and town of publication and publisher in parentheses for books, and finally the page numbers. Titles of journal articles may also be included. Up to twenty authors may be given in a particular reference; where there are more than twenty only the first should be given followed by ‘*et al*’. Abbreviations of the names of periodicals used by Institute of Physics Publishing are usually the same as those given in British Standard BS 4148: 1985. If an author is unsure of an abbreviation and the journal is not given in Appendix B, it is best to leave the title in full. The terms *loc. cit.* and *ibid.* should not be used. Unpublished conferences and reports should generally not be included in the reference list and articles in the course of publication should be entered only if the journal of publication is known. References to preprints should give the title of the preprint and/or preprint number (if relevant). A thesis submitted for a higher degree may be included in the reference list if it has not been superseded by a published paper and is available through a library; sufficient information should be given for it to be traced readily.

## 8. Cross referencing

The facility to cross reference items in the text is very useful when composing articles the precise form of which is uncertain at the start and where revisions and amendments may subsequently be made. When using cross referencing labels are given to elements in the text, for instance sections, figures, tables or equations and the elements may be referred to elsewhere in the text by using the label. When the article is first processed the labels are read in and assigned, e.g. 2.1 for a subsection or (4) for an equation number. When the article is processed a second or subsequent time the label assignments are read in at the start of the file and the correct values given in the text.  $\text{\LaTeX}$  provides excellent facilities for doing cross-referencing and these can be very useful in preparing articles.

### 8.1. References

Cross referencing is useful for numeric reference lists because, if it is used, adding another reference to the list does not then involve renumbering all subsequent references. It is

not necessary for referencing in the Harvard system where the final reference list is alphabetical and normally no other changes are necessary when a reference is added or deleted. Two passes are necessary initially to get the cross references right but once they are correct a single run is usually sufficient provided an `.aux` file is available and the file is run to the end each time. `\cite` and `\bibitem` are used to link citations in the text with their entry in the reference list; if the reference list contains an entry `\bibitem{label}`, then `\bibitem{label}` will produce the correct number in the reference list and `\cite{label}` will produce the number within square brackets in the text. `label` may contain alphabetic letters, or punctuation characters but must not contain spaces or commas. It is also recommended that the underscore character `_` is not used in cross referencing. Thus labels for the form `eq:partial`, `fig:run1`, `eq:dy'`, etc, may be used. When several references occur together in the text `\cite` may be used with multiple labels with commas but no spaces separating them; the output will be the numbers within a single pair of square brackets with a comma and a thin space separating the numbers. Thus `\cite{label1,label2,label4}` would give [1,2,4]. Note that no attempt is made to sort the labels and no shortening of groups of consecutive numbers is done. Authors should therefore try to use multiple labels in the correct order.

The numbers for the cross referencing are generated in the order the references appear in the reference list, so that if the entries in the list are not in the order in which the references appear in the text then the numbering within the text will not be sequential. To correct this change the ordering of the entries in the reference list and then rerun *twice*.

## 8.2. Equation numbers, sections, subsections, figures and tables

Cross references can be made to equation numbers, sections, subsections, figures and tables or any numbered environment and this is a very useful feature when writing a document as its final structure is often not fully defined at the start. Thus a later section can be referred to by a label before its precise number is known and when it is defined there is no need to search back through the document to insert the correct value manually. For this reason the use of cross referencing can save considerable time.

Labels for equation numbers, sections, subsections, figures and tables are all defined with the `\label{label}` command and cross references to them are made with the `\ref{label}` command. The `\label` macro identifies the type of environment it is used in and converts `label` into the correct form for that type of environment, thus `\ref{label}` might give (2.3) for an equation number but 3.1 for a subsection and 2 for a figure or table number.

Any section, subsection, subsubsection, appendix or subappendix command defines a section type label, e.g. 1, 2.2, A2, A1.2 depending on context. A typical article might have in the code of its introduction ‘The results are discussed in section~`\ref{disc}`.’ and the heading for the discussion section would be:

**Table 4.** Alternatives to the normal references `\ref` and the text generated by them. Note it is not normally necessary to include the word equation before an equation number except where the number starts a sentence. The versions producing an initial capital should only be used at the start of sentences.

Reference	Text produced
<code>\eref{&lt;label&gt;}</code>	(<num>)
<code>\Eref{&lt;label&gt;}</code>	Equation (<num>)
<code>\fref{&lt;label&gt;}</code>	figure <num>
<code>\Fref{&lt;label&gt;}</code>	Figure <num>
<code>\sref{&lt;label&gt;}</code>	section <num>
<code>\Sref{&lt;label&gt;}</code>	Section <num>
<code>\tref{&lt;label&gt;}</code>	table <num>
<code>\Tref{&lt;label&gt;}</code>	Table <num>

`\section{Results\label{disc}}`

Labels to sections, etc, may occur anywhere within that section except within another numbered environment. Within a maths environment labels can be used to tag equations which are referred to within the text. An example of an equation with a label and a reference to it is:

```
\begin{equation}
X=a\cos\theta+ b\sin\phi. \label{cossin}
\end{equation}
Equation (\ref{cossin}) ...
```

which produces

$$X = a \cos \theta + b \sin \phi. \quad (13)$$

Equation (13) ...

In addition to the standard `\ref{<label>}` the abbreviated forms given in the table 4 are available for reference to standard parts of the text

## 9. Tables and table captions

Tables are numbered serially and referred to in the text by number (table 1, etc, **not** tab. 1). Each table should have an explanatory caption which should be as concise as possible. If a table is divided into parts these should be labelled (*a*), (*b*), (*c*), etc but there should be only one caption for the whole table, not separate ones for each part.

In the preprint style the tables may be included in the text or listed separately after the reference list starting on a new page.

### 9.1. The basic table format

The standard form for a table is:

```

\begin{table}
\caption{Table caption.}
\begin{indented}
\item[]\begin{tabular}{@{}llll}
\br
Head 1&Head 2&Head 3&Head 4\\
\mr
1.1&1.2&1.3&1.4\\
2.1&2.2&2.3&2.4\\
\br
\end{tabular}
\end{indented}
\end{table}

```

Points to note are:

- (i) The caption comes before the table. It should have a full stop at the end.
- (ii) Tables are normally set in a smaller type than the text. The normal style is for tables to be indented in the same way as equations. This is accomplished by using `\begin{indented} ... \end{indented}` and putting `\item[]` before the start of the tabular environment. Omit these commands for any tables which will not fit on the page when indented.
- (iii) The default alignment of columns should be aligned left and adding `@{}` omits the extra space before the first column.
- (iv) Tables have only horizontal rules and no vertical ones. The rules at the top and bottom are thicker than internal rules and are set with `\br` (bold rule). The rule separating the headings from the entries is set with `\mr` (medium rule).
- (v) Numbers in columns should be aligned on the decimal point; to help do this a control sequence `\lineup` has been defined which sets `\0` equal to a space the size of a digit, `\m` to be a space the width of a minus sign, and `\-` to be a left overlapping minus sign. `\-` is for use in text mode while the other two commands may be used in maths or text. (`\lineup` should only be used within a table environment after the caption so that `\-` has its normal meaning elsewhere.) See table 5 for an example of a table where `\lineup` has been used.

## 9.2. Simplified coding and extra features for tables

The basic coding format can be simplified using extra commands provided in the `iopart` class file. The commands up to and including the start of the tabular environment can be replaced by

```
\Table{Table caption}
```

**Table 5.** A simple example produced using the standard table commands and `\lineup` to assist in aligning columns on the decimal point. The width of the table and rules is set automatically by the preamble.

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
23.5	60	0.53	−20.2	−0.22	1.7	14.5
39.7	−60	0.74	−51.9	−0.208	47.2	146
123.7	0	0.75	−57.2	—	—	—
3241.56	60	0.60	−48.1	−0.29	41	15

**Table 6.** A table with headings spanning two columns and containing notes. To improve the visual effect a negative skip (`\ns`) has been put in between the lines of the headings. Commands set-up by `\lineup` are used to aid alignment in columns. `\lineup` is defined within the `\Table` definition.

Nucleus	Thickness (mg cm <sup>−2</sup> )	Composition	Separation energies	
			$\gamma$ , n (MeV)	$\gamma$ , 2n (MeV)
<sup>181</sup> Ta	19.3 ± 0.1 <sup>a</sup>	Natural	7.6	14.2
<sup>208</sup> Pb	3.8 ± 0.8 <sup>b</sup>	99% enriched	7.4	14.1
<sup>209</sup> Bi	2.86 ± 0.01 <sup>b</sup>	Natural	7.5	14.4

<sup>a</sup> Self-supporting.

<sup>b</sup> Deposited over Al backing.

this also activates the definitions within `\lineup`. The final three lines can also be reduced to `\endTable` or `\endtab`. Similarly for a table which does not fit in when indented `\fulltable{caption} ... \endfulltable` or `\endtab` can be used. L<sup>A</sup>T<sub>E</sub>X optional positional parameters can, if desired, be added after `\Table{caption}` and `\fulltable{caption}`.

`\centre{#1}{#2}` can be used to centre a heading `#2` over `#1` columns and `\crule{#1}` puts a rule across `#1` columns. A negative space `\ns` is usually useful to reduce the space between a centred heading and a centred rule. `\ns` should occur immediately after the `\\` of the row containing the centred heading (see code for table 6). A small space can be inserted between rows of the table with `\ms` and a half line space with `\bs` (both must follow a `\\` but should not have a `\\` following them).

Units should not normally be given within the body of a table but given in brackets following the column heading; however, they can be included in the caption for long column headings or complicated units. Where possible tables should not be broken over pages. If a table has related notes these should appear directly below the table rather than at the bottom of the page. Notes can be designated with footnote symbols (preferable when there are only a few notes) or superscripted small roman letters. The notes are set to the same width as the table and in normal tables follow after `\end{tabular}`, each note preceded by `\item[]`. For a full width table `\noindent` should precede the note rather than `\item[]`. To simplify the



**Table 7.** Control sequences to describe lines and symbols in figure captions.

Control sequence	Output	Control sequence	Output
<code>\dotted</code>	.....	<code>\opencircle</code>	○
<code>\dashed</code>	----	<code>\opentriangle</code>	△
<code>\broken</code>	---	<code>\opentriangledown</code>	▽
<code>\longbroken</code>	— — —	<code>\fullsquare</code>	■
<code>\chain</code>	— . —	<code>\opensquare</code>	□
<code>\dashddot</code>	— . . —	<code>\fullcircle</code>	●
<code>\full</code>	——	<code>\opendiamond</code>	◇

coding `\tabnotes` can, if desired, replace `\end{tabular}` and `\endtabnotes` replaces `\end{indented}\end{table}`.

If all the tables are grouped at the end of a document the command `\Tables` is used to start a new page and set a heading ‘Tables and table captions’.

## 10. Figures and figure captions

Figures may be included in an article as encapsulated PostScript files or using the  $\text{\LaTeX}$  picture environment. Alternatively authors may send in high quality printed versions of their figures (fair copies) and attach copies of the fair copies to each typescript. The fair copies should be in black Indian ink or printing on tracing paper, plastic or white card or paper, or glossy photographs.

Each figure should have a brief caption describing it and, if necessary, interpreting the various lines and symbols on the figure. As much lettering as possible should be removed from the figure itself and included in the caption. If a figure has parts, these should be labelled (a), (b), (c), etc. Table 7 gives the definitions for describing symbols and lines often used within figure captions (more symbols are available when using the optional packages loading the AMS extension fonts).

Unless the figures files are incorporated into the text the captions should be listed at the end of the article.

### 10.1. Figure captions where figure files are not available

The command `\Figures` starts a new page and an unnumbered section with the heading ‘Figure captions’. The captions should then be set with the commands:

```
\begin{figure}
\caption{Figure caption.}
\end{figure}

or more simply

\Figure{Figure caption.}
```

The caption should finish with a full stop and the printed version will be indented as in Institute of Physics Publishing single-column journals.

### 10.2. Inclusion of graphics files

If graphics files are available as encapsulated PostScript (EPS) files (or are created within the  $\text{\LaTeX}$  picture environment) they may be included within the body of the text at an appropriate point using a standard graphics inclusion package. Authors should ensure EPS files meet the following criteria

- the Bounding Box should indicate the area of the figure with a minimum of white space around it and not the dimensions of the page.
- Any fonts used should be from the standard PostScript set (Times, Helvetica, Courier and Symbol).
- Scanned images should be of 600 dpi resolution for line art (black and white) and 150 dpi resolution for grayscale or colour.
- Captions and labels (e.g. Figure 1) should not be included in the EPS file although part letters (e.g. (a)) are acceptable provided they are placed close or within the boundary of the figure.

The precise coding required will depend on the graphics package being used and the printer driver. We use a printer driver compatible with DVIPS but authors should avoid using special effects generated by including verbatim PostScript code within the  $\text{\LaTeX}$  file with specials other than the standard figure inclusion ones.

Using the epsf package figures can be included using code such as:

```
\begin{figure}
\begin{center}
\epsfbox{file.eps}
\end{center}
\caption{Figure caption}
\end{figure}
```

## Appendix A. List of macros for formatting text, figures and tables

**Table A1.** Macros available for use in text. Parameters in square brackets are optional.

Macro name	Purpose
<code>\title[#1]{#2}</code>	Title of article and short title (optional)
<code>\paper[#1]{#2}</code>	Title of paper and short title (optional)
<code>\letter{#1}</code>	Title of Letter to the Editor
<code>\comment[#1]{#2}</code>	Title of Comment and short title (optional)
<code>\topical[#1]{#2}</code>	Title of Topical Review and short title (optional)
<code>\review[#1]{#2}</code>	Title of review article and short title (optional)
<code>\note[#1]{#2}</code>	Title of Note and short title (optional)
<code>\prelim[#1]{#2}</code>	Title of Preliminary Communication & short title
<code>\author{#1}</code>	List of all authors
<code>\article[#1]{#2}{#3}</code>	Type and title of other articles and short title (optional)
<code>\address{#1}</code>	Address of author
<code>\pacs{#1}</code>	PACS classification codes
<code>\pacno{#1}</code>	Single PACS classification code
<code>\ams{#1}</code>	American Mathematical Society classification code
<code>\jl{#1}</code>	Number of journal article submitted to
<code>\submitted</code>	‘Submitted to’ message
<code>\maketitle</code>	Creates title page
<code>\begin{abstract}</code>	Start of abstract
<code>\end{abstract}</code>	End of abstract
<code>\nosections</code>	Inserts space before text when no sections
<code>\section{#1}</code>	Section heading
<code>\subsection{#1}</code>	Subsection heading
<code>\subsubsection{#1}</code>	Subsubsection heading
<code>\appendix</code>	Start of appendixes
<code>\ack</code>	Acknowledgments heading
<code>\References</code>	Heading for reference list
<code>\begin{harvard}</code>	Start of alphabetic reference list
<code>\end{harvard}</code>	End of alphabetic reference list
<code>\begin{thebibliography}{#1}</code>	Start of numeric reference list
<code>\end{thebibliography}</code>	End of numeric reference list
<code>\etal</code>	<i>et al</i> for text and reference lists
<code>\dash</code>	Rule for repeated authors in alphabetical reference list
<code>\nonum</code>	Unnumbered entry in numerical reference list

**Table A2.** Macros defined within `iopart.cls` for use with figures and tables.

Macro name	Purpose
<code>\Figures</code>	Heading for list of figure captions
<code>\Figure{#1}</code>	Figure caption
<code>\Tables</code>	Heading for tables and table captions
<code>\Table{#1}</code>	Table caption
<code>\fulltable{#1}</code>	Table caption for full width table
<code>\endTable</code>	End of table created with <code>\Table</code>
<code>\endfulltab</code>	End of table created with <code>\fulltable</code>
<code>\endtab</code>	End of table
<code>\br</code>	Bold rule for tables
<code>\mr</code>	Medium rule for tables
<code>\ns</code>	Small negative space for use in table
<code>\centre{#1}{#2}</code>	Centre heading over columns
<code>\crule{#1}</code>	Centre rule over columns
<code>\lineup</code>	Set macros for alignment in columns
<code>\m</code>	Space equal to width of minus sign
<code>\-</code>	Left overhanging minus sign
<code>\0</code>	Space equal to width of a digit

## Appendix B. Control sequences for journal abbreviations

**Table B1.** Abbreviations for the IOPP journals.

Macro name	Short form of journal title	Years relevant
\CQG	Class. Quantum Grav.	
\CTM	Combust. Theory Modelling	
\EJP	Eur. J. Phys.	
\IP	Inverse Problems	
\JO	J. Opt.	
\JPA	J. Phys. A: Math. Gen.	
\JPB	J. Phys. B: At. Mol. Phys.	1968–1987
\jpb	J. Phys. B: At. Mol. Opt. Phys.	1988 and onwards
\JPC	J. Phys. C: Solid State Phys.	1968–1988
\JPCM	J. Phys: Condens. Matter	1989 and onwards
\JPD	J. Phys. D: Appl. Phys.	
\JPE	J. Phys. E: Sci. Instrum.	1968–1989
\JPF	J. Phys. F: Met. Phys.	
\JPG	J. Phys. G: Nucl. Phys.	1975–1988
\jpg	J. Phys. G: Nucl. Part. Phys.	1989 and onwards
\MSMSE	Modelling Simul. Mater. Sci. Eng.	
\MST	Meas. Sci. Technol.	1990 and onwards
\NET	Network	
\NL	Nonlinearity	
\NT	Nanotechnology	
\PAO	Pure and Applied Optics	
\PMB	Phys. Med. Biol.	
\PSST	Plasma Sources Sci. Technol.	
\PUS	Public Understand. Sci.	
\QO	Quantum Opt.	
\RPP	Rep. Prog. Phys.	
\SST	Semicond. Sci. Technol.	
\SUST	Supercond. Sci. Technol.	
\WRM	Waves in Random Media	

**Table B2.** Abbreviations for some more common non-IOPP journals.

Macro name	Short form of journal
\AC	Acta Crystallogr.
\AM	Acta Metall.
\AP	Ann. Phys., Lpz
\APNY	Ann. Phys., NY
\APP	Ann. Phys., Paris
\CJP	Can. J. Phys.
\GRG	Gen. Rel. Grav.
\JAP	J. Appl. Phys.
\JCP	J. Chem. Phys.
\JJAP	Japan. J. Appl. Phys.
\JMMM	J. Magn. Magn. Mater.
\JMP	J. Math. Phys.
\JOSA	J. Opt. Soc. Am.
\JP	J. Physique
\JPhCh	J. Phys. Chem.
\JPSJ	J. Phys. Soc. Japan
\JQSRT	J. Quant. Spectrosc. Radiat. Transfer
\NC	Nuovo Cimento
\NIM	Nucl. Instrum. Methods
\NP	Nucl. Phys.
\PF	Phys. Fluids
\PL	Phys. Lett.
\PR	Phys. Rev.
\PRL	Phys. Rev. Lett.
\PRS	Proc. R. Soc.
\PS	Phys. Scr.
\PSS	Phys. Status Solidi
\PTRS	Phil. Trans. R. Soc.
\RMP	Rev. Mod. Phys.
\RSI	Rev. Sci. Instrum.
\SSC	Solid State Commun.
\SPJ	Sov. Phys.–JETP
\ZP	Z. Phys.